Intra-atomic binding energy

Calculation according to the Wave Model, Part 3

Deuterium and Tritium

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1. Introduction

We continue the description devoted to the direct derivation of the binding energy of atoms, begun in Part 1 and 2 of this article [1, 2], which became possible thanks to the key discoveries made in the framework of the Wave model (WM).

At the beginning of this last part of the article, I decided to remind some of the ideas that led us to the creation of the WM. They are directly related to the problem considered here.

As known, particular solutions (potential component)

$$\psi_p = A\sqrt{\frac{\pi}{2\rho}}J_{l+1/2}(\rho)\Theta_{l,m}(\theta)\,\mathrm{Cos}(m\varphi+\alpha) \qquad (1)$$

of the wave equation $\Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0$ describe the location of the nodes of standing waves in a spherical wave field-space. Kinetic nodes, antinodes of standing wave, shifted relative to the potential ones, have the same form; they do not interest us here.

We presented the above solutions for the first time in physics in graphical form, moreover, in such a way as shown in Fig. 1 [3, 4].

Functions $Y_{l,m}(\theta,\varphi)_p = \left|\Theta_{l,m}(\theta)\,\mathrm{Cos}\,m\varphi\right|$ give the spatial *angular coordinates* of the nodes of standing spherical waves (and toroidal vortices-rings, for m=0) on the corresponding radial wave shells described by the radial function $\hat{R}_l(\rho) = A\sqrt{\pi/2\rho}(J_{l+1/2}(\rho)$ (shown at top right in Fig. 1).

Analyzing the obtained solutions, we assigned serial numbers to the *principal potential polar-azimuthal* wave nodes, in the order of increasing their numbers in wave shells, starting from the nodal shell corresponding to the solution for l=1 and m=±1.

Having accomplished this, we found a striking correlation of the number of such nodes in wave shells with atomic numbers Z of elements in the periodic table [3, 4], assuming herewith that each of the nodes of the stable atomic isotopes contains two coupled nucleons.

We also drew attention to the periodic quasi-similarity of the nodal structure (depending on the quantum numbers l and m [5]), which clearly correlates with the periodicity of the chemical and physical properties of the elements grouped in the periodic table.

Ultimately, we understood and completely convinced that solutions (1) describe the nodal structure of wave shells of atoms. Thus, we have come to the discovery of a shell-nodal (molecule-like) structure of atoms that atoms are elementary molecules of nucleons.

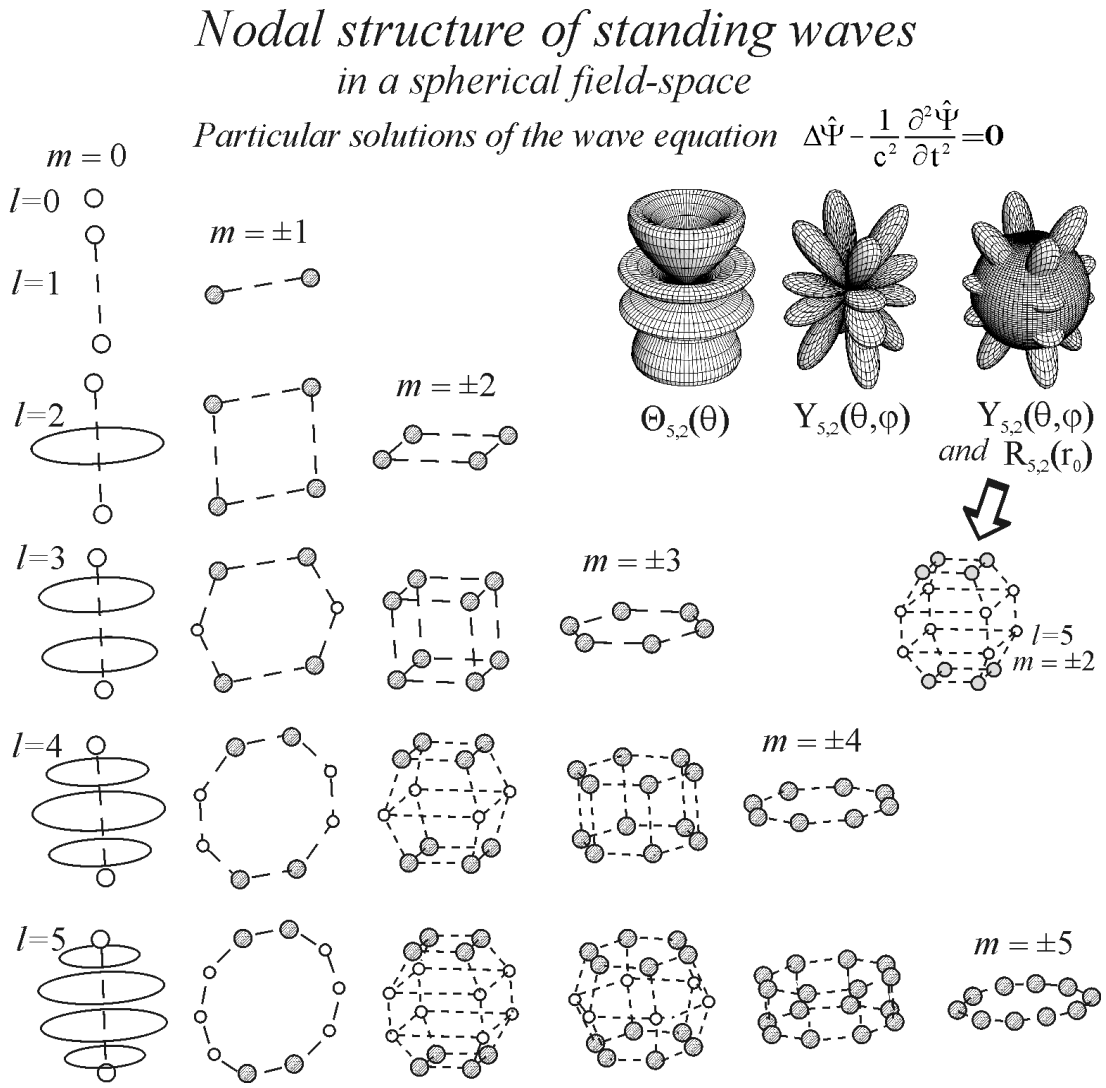

Fig. 1. Particular solutions of the wave equation in spherical polar coordinates.

A graphical representation of the solutions, indicating their relation to the description of the atoms and revealing the primary nature of the observed periodicity of the chemical (and other) properties of the elements of the periodic table, is shown in Fig. 2.

We assigned serial numbers only to the main potential polar-azimuthal nodes (shown in dark circles). Only symbols of atoms having external wave shells, fully complete and half-completed with the given nodes, are indicated in the figure.

Thus, the physical meaning of the solutions (presented in Fig. 1), previously unknown in physics, was discovered in this way. The above solutions showed that atoms are wave formations and their structure is identical to the nodal structure of standing waves in a spherical field.

The nodal structure of each subsequent atom, from the lightest to the heaviest, is different. This is due to the different number of nodes in their external wave shells. Changing the number of nodes characteristic to the outer shell of the previous atom by one more is accompanied by a rearrangement of the spatial disposition of the nodes. As a result, a more complex nodal

structure is formed, different from the previous one, inherent in the subsequent havier individual atom.

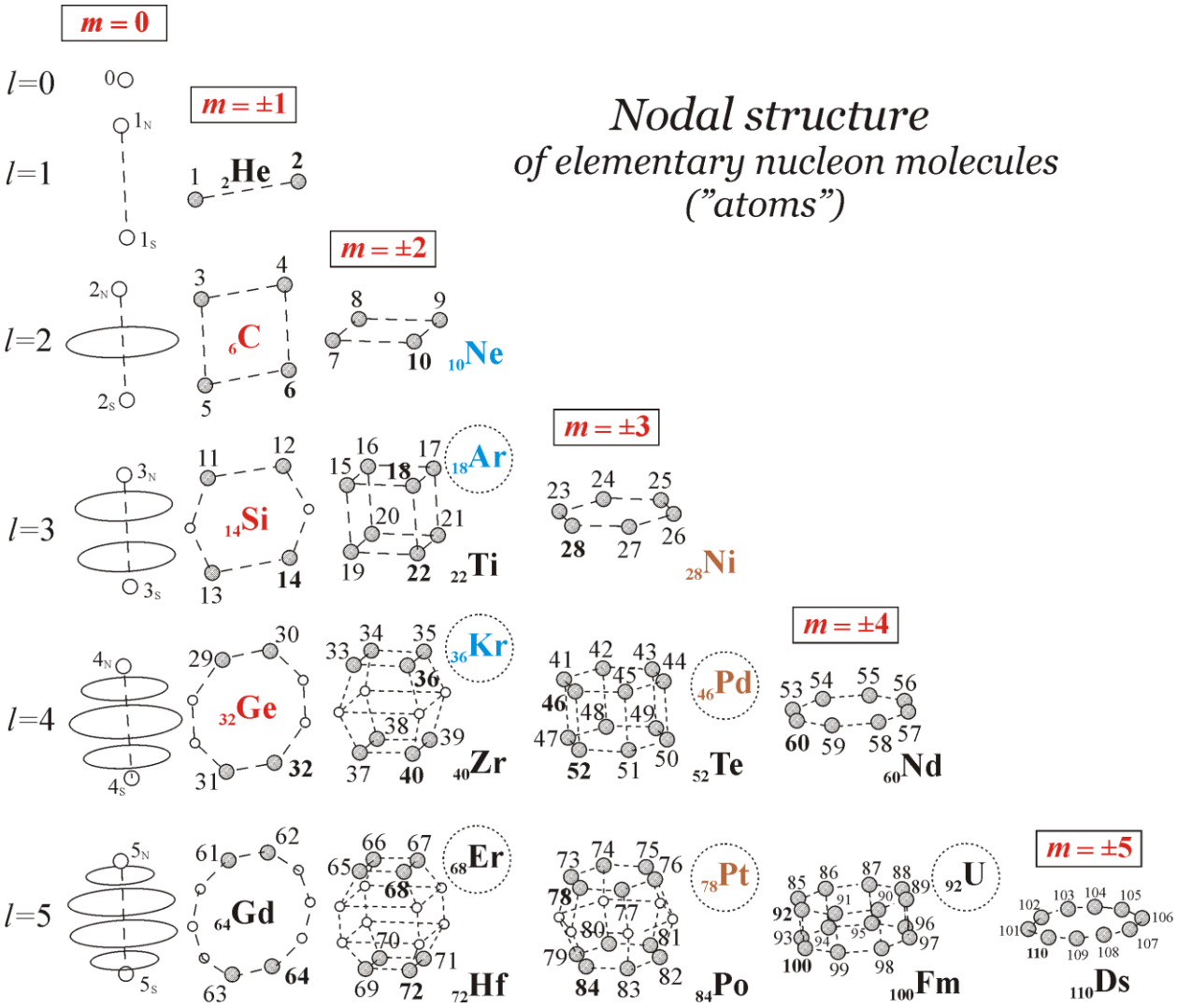

Fig. 2. The first graphical representation of solutions (1), indicating their relation to the description of the nodal structure of atoms, and demonstrating the periodical quasi-similarity of the nodal structure of wave atomic shells.

The collateral (unnumbered, empty) potential polar-azimuthal nodes, depicted by the smallest white circles, first appeared in the external shell of the silicon atom (l = 3, m = ± 1) and further in the external shell of the germanium atom (l = 4, m = ± 1).

Apparently, the phenomenon of semiconductivity of Si and Ge is caused precisely by the presence of such nodes in their external shells: two in Si and four in Ge, respectively.

The external shell of the gadolinium atom (belonging to the same group with C, Si, and Ge) contains *six* such *empty collateral* nodes compared the *four main* nodes *filled* with coupled nucleons. Maybe for this reason gadolinium is characterized by the highest *capture cross section* of *thermal neutron* among all elements. Such a large capture cross-section makes it possible to use gadolinium in controlling a nuclear chain reaction and for protection against neutrons.

Three more atoms, whose *outer* shells have collateral nodes, are $_{40}$Zr, $_{72}$Hf , and $_{84}$Po.

We note here only a few features, which we immediately noticed, starting to analyze the obtained solutions, which we would like to draw the attention of readers.

Sequence numbers 1, 2, 3,. .., 110, which we assigned to the *principal potential polar-azimuthal* nodes, correspond to the *atomic numbers* Z of the elements of the Periodic Table.

Nucleon molecules ("atoms"), symbols of which are indicated in Fig. 2 at the corresponding fully completed and half-completed shells and subshells (with the even number of nodes in them), differ from others (not indicated here) for the above reason by the more equilibrium structure of their external wave shells.

Nucleon molecules ("atoms") having *fully completed l*-shells (from l=1 to l=5) are, respectively:

$$_{2}\mathrm{He},\ _{10}\mathrm{Ne},\ _{28}\mathrm{Ni},\ _{60}\mathrm{Nd},\ _{110}\mathrm{Ds}\quad (l=1, 2, 3, 4, 5)$$

Nucleon molecules ("atoms"), the outer shells of which are <u>*quasi-similar*</u> *fully completed m*-shells (subshells of the *l*-shells), are:

$$_{6}\mathrm{C},\ _{14}\mathrm{Si},\ _{32}\mathrm{Ge},\ _{64}\mathrm{Gd}\quad (m=\pm1)\quad \text{(see Fig. 3)}$$
$$_{22}\mathrm{Ti},\ _{40}\mathrm{Zr},\ _{72}\mathrm{Hf}\quad (m=\pm2)$$
$$_{52}\mathrm{Te},\ _{84}\mathrm{Po}\quad (m=\pm3)$$

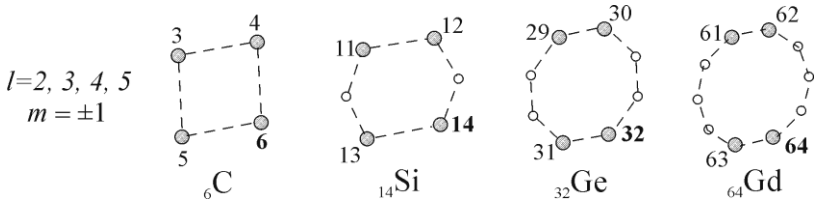

Fig. 3. Outer *quasi-similar* shells of elementary nucleon molecules ("atoms") of the 4th group of the periodic table of elements.

Nucleon molecules ("atoms"), the outer shells of which are <u>*similar*</u> *completed* and *half-completed* (indicated in Fig. 2 in dotted circles) *m*-shells, are as follows:

$$_{10}\mathrm{Ne},\ _{18}\mathrm{Ar},\ _{36}\mathrm{Kr},\ _{68}\mathrm{Er}\quad (m=\pm2)$$
$$_{28}\mathrm{Ni},\ _{46}\mathrm{Pd},\ _{78}\mathrm{Pt}\quad (m=\pm3)$$
$$_{60}\mathrm{Nd},\ _{92}\mathrm{U}\quad (m=\pm4)$$

External nodes of uncompleted external shells and subshells: half-completed (shown above) and partially filled, such as the shells of $_{3}$Li, $_{4}$Be, $_{5}$B or $_{7}$N, $_{8}$O, $_{9}$F, etc. are in equatorial plane (z=0), like the shells of $_{10}$Ne, $_{28}$Ni … Such shells have any-fold symmetry, including forbidden by mathematical laws of crystallography. This follows from corresponding noninteger solutions, which describe intermediate states [4, 5].

So, analyzing solutions (1), we came to the discovery of the molecule-like structure of atoms. Atoms are elementary molecules of nucleons whose structure is identical to the nodal structure of standing waves in a spherical wave field-space.

All details about the solutions of the wave equation, including presented in the form shown above and in other forms, are considered in [3, 4]; some details can also be found, for example, in [5].

In our further consideration, we will refer to Fig. 2.

2. The g-lepton structure of nucleons

In accordance with the basic axiom of dialectical philosophy and its logic (dialectics), all objects and phenomena in the Universe, including particles at all its levels, have the wave nature. This axiom is in the basis of the WM.

Accordingly, the structure of all particles should be described by well-developed methods of the physics of waves and, in particular, by the general ("classical") wave equation $\Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0$. This equation admits a particular solution, which is a complex (potential-kinetic) function of three variables, $\hat{\psi}(\rho,\theta,\varphi) = A\hat{R}_l(\rho)\Theta_{l,m}(\theta)\hat{\Phi}_m(\varphi)$. The latter has the form of a product of three separate functions: *radial*, *polar* and *azimuthal*, dependent on one variable, $\rho = kr$, θ, and φ, respectively. This solution describes the standing waves (location of nodes and antinodes) in a spherical space, and, as we have shown, the shell-nodal structure of "atoms".

Accepting the above axiom, it is natural to assume that solution (1) describe also the internal shell-nodal structure of particles of the lower (subatomic) level, in particular, the internal shell-nodal structure of nucleons (protons and neutrons).

Really, in two parts of the article [6, 7], we have shown that nucleons (protons and neutrons) most likely consist of g-leptons.

By analogy with the structure of elementary nucleon molecules ("atoms") of the nucleon level, *nucleons*, at the g-*lepton level* of the Universe, have a shell-nodal structure *similar* to the shell-nodal structure of a *silicon* atom with atomic number 14 (having 14 nodes, according to solutions of the above wave equation, filled with coupled nucleons).

Indeed, let the mass of the g-lepton in units of the mass of the electron m_e be exactly a multiple of a quarter of the fundamental period-quantum $\Delta=2\pi\lg e$ [8],

$$m_g = (1/4)2\pi\lg e\cdot 10^2\, m_e = 68.21881769\, m_e. \qquad (2)$$

The masses of the proton and neutron in the same units are equal, respectively, to

$$m_p = 1836.1526675\, m_e \qquad \text{and} \qquad m_n = 1838.683645\, m_e. \qquad (3)$$

Therefore, due to the relation

$$m_n / m_g = 26.95273397 \qquad (4)$$

and taking into account a certain value of the binding energy of g-leptons caused by the mass defect phenomenon, influenced on the resulting mass of nucleons, it is clear that the mass number of nucleons at the g-lepton level should be somewhat larger than 27.

Based on this, we hypothesized that protons and neutrons represent at the g-lepton level, respectively, two stable isotopes of the silicon atom, ${}^{28}_{14}Si$ and ${}^{29}_{14}Si$, having the same shell-nodal structure.

The nodal structure of ${}^{28}_{14}Si$ is presented separately in Fig. 4, in full agreement with solutions of the wave equation, depicted in Fig. 2.

The second stable isotope ${}^{29}_{14}Si$ differs from ${}^{28}_{14}Si$ by the presence of yet one nucleon, which is located in the central polar potential-kinetic ("0") node.

The relative disposition in a neutron, considering as an analogue of the stable isotope of the silicon atom ${}_{14}Si$, of four wave spherical shells (I - IV) and the spatial arrangement in them of

the 14 potential polar-azimuthal nodes, filled with coupled g-nucleons (numbered dark circles), are shown in Fig. 5.

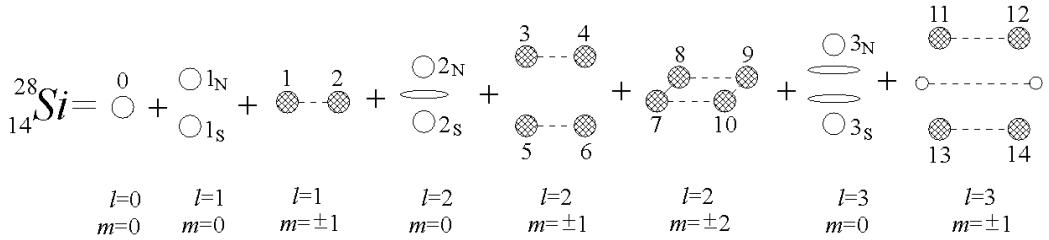

Fig. 4. The symbolic representation of the components of the shell-nodal structure of the $^{28}_{14}Si$ silicon atom in accordance with the solutions shown in Fig. 2.

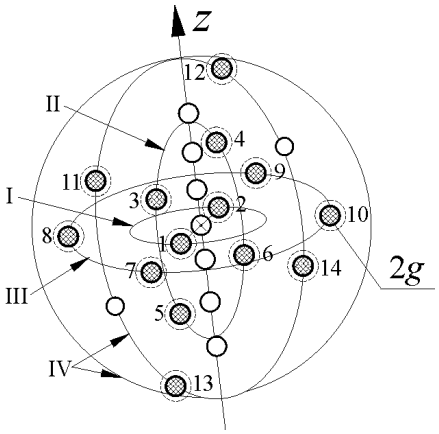

Fig. 5. The g-nucleon shell-nodal structure of the neutron (*n*), identical with the shell-nodal structure of the silicon atom $_{14}$Si, following from solutions of the wave equation,

We are considering a neutron having a structure in the form, shown in Fig. 5, as one of the isotopes of the simplest hydrogen atom – protium $^{1}_{1}H$, that is, as an analog of the stable silicon isotope $^{29}_{14}Si$.

Compared to the proton ($^{28}_{14}Si$ analog), the neutron additionally contains one g-lepton in the central polar node (marked with a cross in Fig. 5).

Being bound in an atom, a neutron is stable, but in a free state it decays during $\tau = 1000\,s$ into a proton, electron, and neutral g-lepton (antineutrino $\tilde{\nu}$, according to nuclear physics):

$$n \to p + e^{-} + g\,. \qquad (5)$$

The shell-nodal structure of $_{14}Si$ (Fig. 4 and 5) is more complex than the shell-nodal structure of carbon $_{6}C$ [2]. Silicon $_{14}Si$ has by two shells (III and IV) and, accordingly, eight nodes more (at $l = 2, m = \pm 2$ and $l = 3, m = \pm 1$) than carbon $_{6}$C.

The *first inner* shell (I) having two polar-azimuthal nodes (1, 2) is the shell of the *helium* atom [1]. The *second inner* shell (II) having four polar-azimuthal nodes (3, 4, 5, 6) is the outer shell of the *carbon* atom. The *third inner* shell (III) is the outer shell of the *neon* atom. Shell IV is the outer shell characteristic of a silicon atom.

In accordance with the shell-nodal atomic model, the unique (specific) structure of the outer shells mainly determines the properties of individual atoms, distinguishing them from each other. The outer shell of $_{14}Si$ has two *collateral nodes* that are not filled with nucleons in stable silicon isotopes.

Silicon is the first element of the periodic table in which, with increasing atomic number z, collateral nodes appeared (unnumbered in Fig. 2 and other figures). Amplitudes of polar-azimuthal functions (see the image in the upper right corner in Fig. 1), determining their spatial location on radial shells, are significantly less than the amplitudes of the corresponding functions that determine the position of the principal nodes numbered in the figures. Apparently, for this reason, collateral nodes are less suitable places for the equilibrium arrangements of nucleons in them.

The above structural feature of silicon (the presence of empty collateral nodes) provides the possibility of movement in its inner space, not just particles, which are much smaller than nucleons, but also the movement of the nodal nucleons themselves. The so-called hole conductivity in semiconductors, apparently, is due to the existence of such nodes.

From the point of view of the shell-nodal model, a neutron having the structure presented above is an isotope of the simplest hydrogen atom – protium ${}^{1}_{1}H$. Accordingly, it has the magnetic moment, the negative value of which, according to the CODATA [9], is

$$\mu_n = -0.96623640(23) \cdot 10^{-26}\ J \cdot T^{-1} \qquad (6)$$

This is approximately 1.46 times less in absolute value of the positive magnetic moment that the proton has.

As a system of a proton and an electron, the structure of the neutron with the surrounding field, following the WM, looks like it is conditionally shown in Fig. 6.

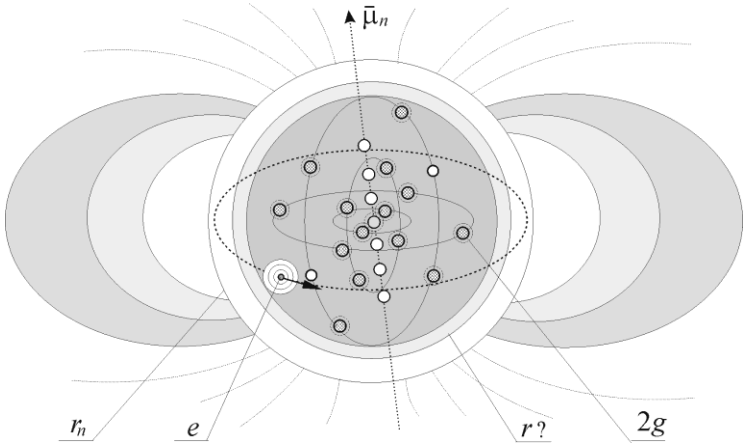

Fig. 6. Neutron as an analogous of the silicon atom ${}^{29}_{14}Si$ with the surrounding field.

In this figure, r_n is the radius of the outer shell of the neutron; *e* is an electron; *r*? is the inner radius of the outer neutron shell; 2*g* is the principal *potential* polar-azimuthal node filled with two coupled *g*-leptons; $\bar{\mu}_n$ is the magnetic moment of the neutron. The 29th g-lepton is located in the central polar *potential-kinetic* node.

According to the mass defect formula, the binding energy of a *proton*, consisting of 28 g-leptons, is equal to

$$\Delta E(p) = c^2 \Delta m = c^2 (28 m_g - m_p) = 62.79769638\, MeV\ , \qquad (7)$$

so the binding energy per *one* g-lepton is

$$\varepsilon(p) = \Delta E(p) / A(p) = 2.242774871 MeV\ , \qquad (8)$$

where $A(p) = 28$ is the mass number of the proton at the g-lepton level.

Corresponding binding energies for the *neutron* ($A(n) = 29$) are

$$\Delta E(n) = c^2(29m_g - m_n) = 71.36715712\,MeV\,, \tag{9}$$

$$\varepsilon(n) = \Delta E(n)/A(n) = 2.460936452\,MeV\,. \tag{10}$$

Now, relying on the shell-nodal g-lepton structure of nucleons, we can proceed to the derivation of the binding energy of deuterium and tritium.

The *main (dominant) component* of these energies is the energy of *strong bonds* between all pairs of coupled g-nucleons, approaching to each other, belonging to g-nucleon nodes of two (during the formation of deuterium) and three (during the formation of tritium) contacting nucleons. The derivation of the above energies became possible due to the discovery (within WM) of the Universal Law of Central Exchange and the corresponding formulas arising from the law [4].

3. Calculation of the binding energy of deuterium ${}^{2}_{1}H$

With strong binding of two nucleons, a neutron and a proton, a stable isotope of a hydrogen atom, deuterium ${}^{2}_{1}H$, is formed.

The g-lepton (shell-nodal) structure of both nucleons is identical. In the process of binding, the nucleon spaces penetrate each other, so that the partial overlap of the spherical shells of both nucleons occurs, as shown in Fig. 7.

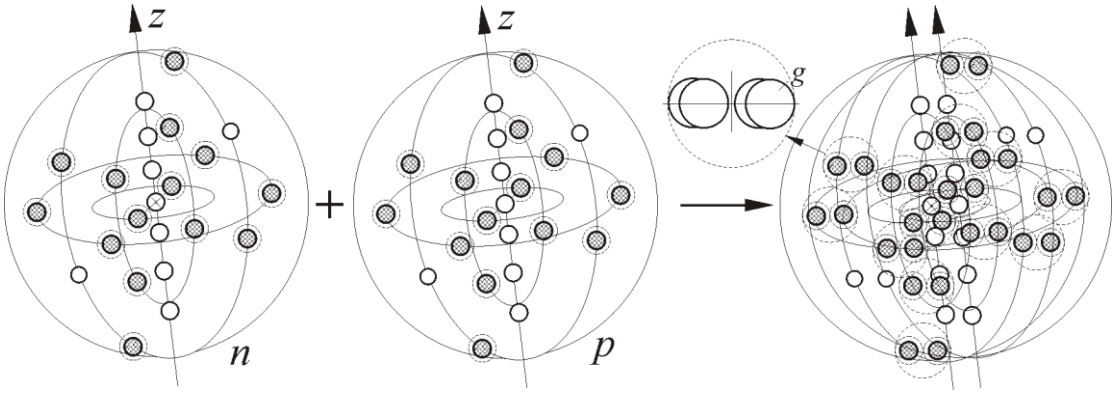

Fig. 7. The formation of deuterium from nucleons of the shell-nodal structure.

With this, all g-lepton nodes of one nucleon and corresponding nodes of another nucleon (both filled with coupled g-leptons), draw together at the distance *r* determined by solutions (1) of the wave equation.

As a result of the binding of the approaching pairs of coupled g-leptons, 14 helium-like structures [1] (depicted by dashed circles in Fig. 7) are formed. The distance *r* between the g-lepton nodes in them (Fig. 8) is determined by the roots z_{mn} of the Bessel functions [10],

$$z_{m,n} = kr\,. \tag{11}$$

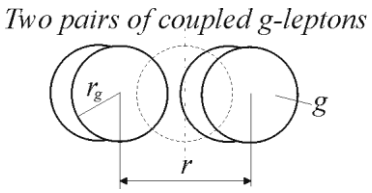

Fig. 8. The helium-like structure formed on the basis of binding of two pairs of coupled g-leptons.

An unknown value in this expression is the wave number *k* equal to the inverse value of the wave radius $\lambda\!\!\!^{-}$,

$$k = 1/\lambda\!\!\!^{-}. \tag{12}$$

The wave radius $\lambda\!\!\!^{-}$ determines the characteristic radii (11) of elementary spherical and cylindrical surfaces determined by the roots of the Bessel functions with zero and extremal values. At the *atomic* and *subatomic* level, $k = 1/\lambda\!\!\!^{-}_e$, where $\lambda\!\!\!^{-}_e = 1.603886538 \cdot 10^{-8}\, cm$ follows from the DM [11, 12].

The wave radius $\lambda\!\!\!^{-}_e$ is responsible for the arrangement of *nucleons* in atoms, and hence in crystals, molecules, *etc*., at the strictly defined absolute distances. Indeed, a double value of the wave radius, wave diameter, is equal to the average value of the constants of crystal lattices:

$$2\lambda\!\!\!^{-}_e \approx 3.2 \cdot 10^{-8} cm, \tag{13}$$

Thus, the wave radius at the *nucleon* level $\lambda\!\!\!^{-}_n$ is equal to (coincides with) the fundamental wave *radius of exchange* at the atomic and subatomic levels $\lambda\!\!\!^{-}_e$,

$$\lambda\!\!\!^{-}_n = \lambda\!\!\!^{-}_e = c/\omega_e, \tag{14}$$

which defines the principal parameters of atomic spaces. Herewith,

$$\omega_e = e/m_e \tag{15}$$

is the fundamental *frequency of exchange* at the atomic and subatomic levels (the frequency of the so-called “electrostatic field”).

The inner spherical space of a *nucleon*, like the inner spherical space of an *atom*, is a system of wave shells, whose *relative* radii are determined by the same roots of the Bessel functions, $\rho = kr = z_{m,n}$. The wave shells of nucleons with intrinsic nodes, in which g-particles are localized, form a superfine discrete structure of atoms.

Solutions (1) of the wave equation give the *relative* values of the radii ρ and the *relative* value of the wave radius $\lambda\!\!\!^{-}$ for spaces of different levels. Obviously, the inner spaces of nucleons, which are spaces of a more discrete (g-*lepton*) level, have a wave radius $\lambda\!\!\!^{-}_g$ smaller than the radius $\lambda\!\!\!^{-}_e$ characteristic for nucleon-level spaces.

The a*bsolute* value of $\lambda\!\!\!^{-}_g$ should be sought from the general conditions for wave processes occurring at different levels.

We will determine $\lambda\!\!\!^{-}_g$ by the scale analogy, which exists between wave processes at any levels and, in particular, between those that occur at the levels of nucleons and g-leptons. The fact is that the fundamental relations, existing between the main wave parameters at both levels, are preserved.

One of such fundamental relations is the relation that exists between the *radius r* of the wave spherical shell of a particle and the fundamental *wave radius* $\lambda\!\!\!^{-}$ of exchange of the particle with other particles and the surrounding field.

For the *proton*, the theoretical *radius* of its *wave shell* obtained by the formula following from the DM [11, 12] , where $\varepsilon_0 = 1 g \cdot cm^{-3}$ and $\varepsilon_r = 1$, at the condition $(k_e r_p)^2 << 1$, is

$$r_p(th) = (m_p / 4\pi\varepsilon_0)^{1/3} = 0.510578616 \cdot 10^{-8}\, cm. \tag{16}$$

The fundamental *wave radius* at the atomic and subatomic levels is

$$\lambda\!\!\!^{-}_e = c/\omega_e = 1.603886538 \cdot 10^{-8}\, cm. \tag{17}$$

The *ratio* of the above fundamental parameters characteristic of the *proton* is, with sufficient accuracy, equal to the fundamental constant π,

$$\lambda\!\!\!^{-}_{e} / r_p(th) = 3.141311617 \approx \pi . \qquad (18)$$

This ratio shows that the *wave radius* $\lambda\!\!\!^{-}_{e}$, in value, is a half of the length of the equatorial circumference of the wave spherical shell of a proton.

Obviously, the same ratio should be valid for the *radius* r_g of the wave spherical shell of g-*lepton*, and the *wave radius* $\lambda\!\!\!^{-}_{g}$, characteristic for particles of the g-lepton level, so that we have the right to assume that

$$\lambda\!\!\!^{-}_{g} / r_g(th) = \pi . \qquad (19)$$

Hence, for

$$r_g(th) = (m_g / 4\pi\varepsilon_0)^{1/3} = 0.170370509 \cdot 10^{-8}\ cm \qquad (20)$$

where $m_g = 6.214420763 \cdot 10^{-26}\ g$, the wave radius of the g-lepton level $\lambda\!\!\!^{-}_{g}$ is

$$\lambda\!\!\!^{-}_{g} = \pi r_g(th) = 0.534 \cdot 10^{-8}\ cm . \qquad (21)$$

We see that the value of $\lambda\!\!\!^{-}_{g}$ is close to the Bohr radius $r_0 = 0.529 \cdot 10^{-8}\ cm$. It is quite possible that the more accurate derivations will lead to the complete coincidence of these values, that is, to the equality

$$\lambda\!\!\!^{-}_{g} = r_0 . \qquad (21a)$$

Indeed, we cannot exclude the equality of the above parameters, which both are the basic parameters of the wave sphere atomic space.

Hence, taking the root of Bessel functions, $z_{m,n} = y_{0,1} = 0.89357697$, as in the case of the helium atom, we arrive at the following distance *r* (see Fig. 7) between two pairs of g-leptons in coupling nucleons, consisting of g-leptons:

$$r = y_{0,1}\lambda\!\!\!^{-}_{g} = 0.477 \cdot 10^{-8}\ cm . \qquad (22)$$

This means that wave spherical shells of two H-atoms in the deuterium ${}^{2}_{1}H$ are partially overlapped as is shown in Fig. 9 (where $r_p = 0.51 \cdot 10^{-8}\ cm$ (16)). Centers of masses of two constituent H-atoms are at the distance $r = 0.477 \cdot 10^{-8}\ cm$, which is some less than the Bohr radius, $r < r_0$.

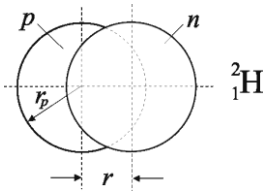

Fig. 9. The relative disposition of two nucleons in the deuterium atom ${}^{2}_{1}H$; $r < r_0$.

At such conditions (distance *r* of the value (22)), the binding energy of two internodal g-leptons (by one from each interacting nodes) in helium-like structure (Fig. 7, 8) is

$$E_g = \frac{q_g^2}{8\pi\varepsilon_0 r} = 0.070246848 \cdot 10^{6}\ eV , \qquad (23)$$

where

$$q_g = m_g \omega_e = 1.161576228 \cdot 10^{-7}\ g \cdot s^{-1}$$

is the exchange charge of the g-lepton, which is an elementary quantum of exchange at the atomic (including interatomic, molecular) and subatomic (including g-lepton) levels. Recall, at these levels, the fundamental frequency of exchange is ω_e.

In accordance with the shell-nodal g-lepton-structure, the proton has 28 g-leptons (in 14 nodes, each of which is filled with 2 coupled g-leptons). The neutron, in comparison with the proton, has one more g-lepton located in the central polar node (see Fig. 4 and 5).

Thus, due to the fact that all g-leptons participate in the exchange (interaction), we have 28.5 pairs of interacting g-leptons in ${}^{2}_{1}H$. Hence, the total binding energy, associated with the internodal exchange (interaction) of all g-leptons belonging to two interacting nucleons, is

$$E_{g,exch} = 28.5 \cdot E_g = 2.002\,MeV . \qquad (24)$$

The obtained value is close to the known value for the deuteron binding energy calculated by the mass defect formula, $E_D = c^2 \Delta m = 2.224\,MeV$. Energy $E_{g,exch}$ (24) is the main (1*st*) but not the only component of the total binding energy of ${}^{2}_{1}H$ (as in the case of the binding energies of helium and carbon atoms considered earlier [1, 2]).

Following the analogy between wave processes at the two levels under consideration (nucleon and g-lepton), it is necessary also to take into account (2*nd*) the energy of coupling of two g-leptons in their nodes and (3*rd*) the binding energy of g-lepton nodes with the shells in which these nodes are located.

However, we will not derive the remaining (2*nd* and 3*rd*) constituents here. The derivation of the *third* component, we have already shown for helium and carbon atoms [1, 2].

Therefore, it will be quite enough to get here an approximate estimate of the two other constituents of the binding energy mentioned above, based on the identity that exists between the *nodal* structure of *helium* and the *nodal helium-like* structure of a system of two pairs of *coupled* g-leptons (Fig. 8).

The following ratio, $E_{He} / E_D = 12.72482$, exists between the total binding energy of helium ${}^{4}_{2}He$, $E_{He} = 28.3\,MeV$, and its second constituent, the binding energy of coupled nucleons in its nodes, $E_D = 2.224\,MeV$ (deuterium binding energy).

Let us assume that the same ratio keeps and for the corresponding energies of the g-lepton helium-like structure shown in Fig. 8. In this case, because the total binding energy of a deuteron is equal to 2.224 *MeV* (according to the formula $E_D = c^2 \Delta m$), the binding energy of all 28 g-lepton helium-like structures ("deuterons") in all g-lepton nodes should be

$$E_{g(2)} = E_D / 12.72482 = 0.175\,MeV . \qquad (25)$$

And the binding energy per one g-lepton "deuteron" is

$$\varepsilon_{2g} = 6.25\,keV . \qquad (26)$$

Hence, finally, we arrive at the following binding energy of ${}^{2}_{1}H$:

$$E({}^{2}_{1}H) = E_{g,exch} + E_{g(2)} + E_{g(3)} = 2.177\,MeV + E_{g(3)} . \qquad (27)$$

Obviously, the contribution of the *third* constituent $E_{g(3)}$, corresponding to the binding energy of all 28 g-nodes with their wave spherical shells will be less than the contribution of the second constituent estimated above.

Therefore, we assume that after adding of $E_{g(3)}$ to the total energy we will closer approach to the value of 2.224 *MeV* that follows from the mass defect formula $E_D = c^2 \Delta m$.

4. Calculation of the binding energy of tritium ${}^3_1H$

Let us proceed now to the derivation of the binding energy of tritium. The shell-nodal structure of three joined g-lepton nodes in tritium (belonging to three interacting nucleons), on the g-lepton level, recalls the nodal structure of helium isotope ${}^6_2He$ (Fig. 10).

Appearance of two coupled g-leptons in the central polar node slightly changes (increases) the former equilibrium distance *r* existed between outmost pairs of g-leptons in the g-lepton helium-like structure shown in Fig. 8.

The nearest to the $r = 0.477 \cdot 10^{-8}\, cm$ equilibrium distance between g-lepton nodes, admitted by solutions of the wave equation, is the distance equal to the wave radius of the g-lepton level, $\lambda\!\!\!^{-}_g = 0.534 \cdot 10^{-8}\, cm$ (21). Therefore, we accept this value of the distance between the outermost g-lepton nodes in tritium (Fig. 10) for further calculations, so that we have

$$r = \lambda\!\!\!^{-}_g = 0.534 \cdot 10^{-8}\, cm, \qquad r_1 = r_2 = \lambda\!\!\!^{-}_g / 2 = 0.267 \cdot 10^{-8}\, cm. \tag{28}$$

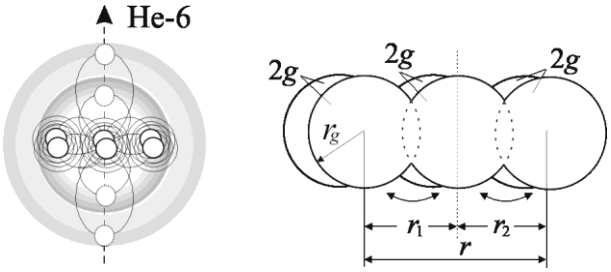

Fig. 10. The nodal structure of helium isotope ${}^6_2He$, and the local helium-like structure formed under the joining of three g-lepton nodes in tritium considered as the *p-n-n* (proton-neutron-neutron) system.

We also assume that the exchange interaction in the presented structure exists between every two partially overlapped pairs as is shown conditionally by two arrows in Fig. 9.

The main constituent of the binding energy in this case, the energy of internodal exchange between two nearest nodes, is

$$E_g = \frac{q_g^2}{8\pi\varepsilon_0 r_1} = 0.140493696\, MeV. \tag{29}$$

Hence, the total binding energy of internodal exchange, with allowance for all g-lepton bonds in tritium, is

$$E_{g,exch} = N_{g,bonds} \cdot E_g = 8.07838752\, MeV, \tag{30}$$

where $N_{g,bonds} = 57.5$ is the number of internodal g-lepton bonds (*p-n* and *n-n*, 28.5+29) in tritium consisted of two neutron and one proton.

According to equality (25), the second constituent that took into account the binding energies of all g-lepton pairs in the nodes of tritium, is

$$E_{g(2)} = N_{g-nodes} \cdot E_{2g} = 0.2625\, MeV, \tag{31}$$

where $N_{g-nodes} = 3 \times 14 = 42\,nodes$ is the number of completed polar-azimuth g-lepton nodes (or the number of coupled g-leptons).

Without the smallest in value contribution of the third constituent $E_{g(3)}$ (related to the binding energy of g-lepton nodes with the shells of their localization), we obtain finally the following magnitude

$$E(^3_1H) = E_{g,exch} + E_{g(2)} = 8.34088752\,MeV\,. \quad (32)$$

For comparison, the binding energy of tritium, originated from the formula on mass defect, is

$$\Delta E = c^2 \Delta m = 8.481821 MeV\,. \quad (33)$$

Thus, we have an approximate coincidence in the resulting data obtained by two ways, different of principle.

5. Conclusion

The capabilities of the modern nuclear model of atoms are limited by the description of the observed phenomena.

The real picture of how the main constituents, nucleons and electrons, are located inside atoms is unknown. It is an insoluble problem in principle, as long as the nuclear model exists in physics. According to the latter, all nucleons in an atom are concentrated in its center, in a small nucleus, in the form of a dense heap of very small and extremely dense (averaging about 2.3×10^{14} g/cm^3) balls. Therefore, the binding energy of atoms is determined in physics solely indirectly - by the formula of mass defect.

Our research over the past few decades has revealed the internal structure of atoms. They showed that atoms have a nuclear-free molecule-like structure. To date, there is a lot of evidence confirming the validity of this discovery, arising, as we have shown, from solutions of the wave equation. The molecular-like structure of atoms (non-nuclear) explains many phenomena, which are impossible to explain in principle in the framework of the nuclear structure of atoms.

Due to the fact that the molecule-like structure of atoms is more adequate to reality, it became possible (for the first time in physics) to directly calculate the binding energy of atoms that was demonstrated by the example of helium, carbon, deuterium and tritium atoms, in three parts of this article.

At the end, we can add the following.

Solutions of the wave equation are applicable to the field-spaces for all levels of the Universe, including levels of spaces of various elementary particles. Therefore, the following assumption makes sense.

In accordance with the dynamic model of elementary particles and the dialectical concept of infinite divisibility of particles [6, 7], it can be assumed that g-lepton, judging by its reference mass m_g=68.22m_e, is a complex atom-molecule of the electron level (*e*-level) with an ordinal number $z \approx 32$.

Such an atom-molecule, having a *mass number more* than 68 in a hypothetical *periodic table* of particles of the *e-level*, corresponds to the $_{32}$Ge germanium atom in *Mendeleev's* periodic table. "*More*" than 68, because we must take into account the binding energy of the constituent particles, electrons, that is, a mass defect. Mass numbers of stable isotopes of germanium are in the range of 70-76.

Interestingly, the *germanium* atom, an analog of the g-*lepton* at the *e-level*, and the *silicon* atom, which is an analog of *nucleons* at the g-*lepton level*, are in the *same* 4th *group* of the periodic table.

Thus, we can say that all *elementary particles* ultimately *consist* of *electrons*. And the mass *spectrum* of the vast variety of *e*-class particles, ranging from the *electron* to the g-*quantum* also exists in nature.

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