## Chapter 3

# AN ANALYSIS OF THE BASIC CONCEPTS OF QUANTUM MECHANICS AND NEW (DIALECTICAL) SOLUTIONS FOR THE FIELD OF A STRING AND H-ATOM 

## 1. Schrödinger's equation and the myths of quantum mechanics

Schrödinger's equation appeared in the years of a wild bloom of formalism, which was represented, first of all, by positivism, machism, pragmatism, and other trends of philosophical thought, denying the objective world. Arbitrary mathematical constructions, on the basis of a free game of notions, were the characteristic result of these philosophical currents for physics. By virtue of this, a reasonable logic in such constructions (theories-myths) was almost absent or insignificant. Schrödinger, a representative of those years, has designed his equation, exactly following the spirit of aforementioned ideological trends in physics.

Nevertheless, we should give Schrödinger his due because the positivistic style did not satisfy his. He had a propensity for actual realism and restrainedly regarded to new fashion trends - the physical mathematical advance-guard. But he was under the influence of that time.

In the 1920's, Schrödinger's equation began to be regarded as the major achievement of scientific thought. It became the basis for lectures on atomic physics in universities. The common opinion is that Schrödinger's equation proved its validity by the conformity of its solutions with the vast amount of experimental data and the co-ordination with general physical notions. However, all this is only emotional judgements.

Now, at the beginning of the $21^{\text {th }}$ century, let us look at the events of those years from the point of view of dialectics and give an objective estimation of the past. Unfortunately, the mathematical model, put forward by Schrödinger, is cur-
rently represented in the form of generalized and extended equations, including the relativistic invariant of them.

In the initial variant, Schrödinger's equation had the following form

$$
\begin{equation*}
\Delta \Psi+\frac{2 m}{\hbar^{2}}\left(W+\frac{e^{2}}{4 \pi \varepsilon_{0} r}\right) \Psi=0 \tag{1.1}
\end{equation*}
$$

Its structure had a quite artificial character and rested upon the operator and variational methods.

The wave $\Psi$-function, satisfying the wave equation (1.1), is represented as

$$
\begin{equation*}
\Psi=R(r) \Theta(\theta) \Phi(\varphi) T(t)=\psi(r, \theta, \varphi) T(t) \tag{1.1a}
\end{equation*}
$$

where $\psi(r, \theta, \varphi)=R(r) \Theta(\theta) \Phi(\varphi)$ is the complex amplitude of the wave function, because

$$
\Phi_{m}(\varphi)=C_{m} e^{ \pm i m \varphi}
$$

The multiplicative form of the amplitude $\psi$-function allows dividing of Schrödinger's equation (1.1) into the equations of the radial $R(r)$, polar $\Theta_{l m}(\theta)$, and azimuth $\Phi_{m}(\varphi)$ functions:

$$
\begin{align*}
& \frac{d^{2} R}{d r^{2}}+\frac{2}{r} \frac{d R}{d r}+\left(\frac{2 m}{\hbar^{2}}\left(W+\frac{Z e^{2}}{4 \pi \varepsilon_{0} r}\right)-\frac{l(l+1)}{r^{2}}\right) R=0  \tag{1.2}\\
& \frac{d^{2} \Theta_{l, m}}{d \theta^{2}}+\operatorname{ctg} \theta \frac{d \Theta_{l, m}}{d \theta}+\left(l(l+1)-\frac{m^{2}}{\sin ^{2} \theta}\right) \Theta_{l, m}=0  \tag{1.2a}\\
& \frac{d^{2} \Phi_{m}}{d \varphi^{2}}=-m^{2} \Phi_{m} \tag{1.2b}
\end{align*}
$$

An equation for the time component of $\Psi$ is

$$
\begin{equation*}
\frac{d^{2} T}{d t^{2}}=-\omega^{2} T \tag{1.2c}
\end{equation*}
$$

The simplest solution of Eq. (1.2c) defines the function of time of the form $T=e^{ \pm i \omega t}$. The last three equations were known in a theory of wave fields. Hence, these equations presented nothing new. Only the radial equation (1.2) was a new one. Its solution turned out to be the divergent one. However, Schrödinger together with H. Weyl (1885-1955, German mathematician), contrary to the logic and all experience of theoretical physics, artificially cut off the divergent power series of the radial function $R(r)$ at a $\kappa$-member. This allowed them to obtain the radial solutions, which, as an effect of the cut off operation, actually are the fictitious solutions. For hydrogen-like atoms, the radial function has the following form

$$
\begin{equation*}
R_{\kappa l}=N_{\kappa l} e^{-\rho /(\kappa+l+1)}\left(\frac{2 \rho}{\kappa+l+1}\right)^{l} L_{\kappa}^{2 l+1}\left(\frac{2 \rho}{\kappa+l+1}\right), \quad \rho=\frac{r}{a} \tag{1.3}
\end{equation*}
$$

where $L_{\kappa}^{2 l+1}\left(\frac{2 \rho}{\kappa+1+l}\right)$ is Laguerre polynomial of the power $\kappa$. At that the power $\kappa$ is simultaneously the parameter of cutting off of the divergent series, $a=\frac{a_{0}}{Z}$ is the large half-axis of the orbit, $a_{0}$ is Bohr radius, and

$$
\begin{equation*}
N_{\kappa l}=\frac{2}{(\kappa+1+l)^{2}} \sqrt{\frac{\kappa!}{(\kappa+1+2 l)!}} \cdot\left(\frac{1}{a}\right)^{3 / 2} \tag{1.4}
\end{equation*}
$$

is the normalizing multiplier.
Since the radial equation (1.2) contains the energy of interaction of the electron with the nucleus $-\frac{Z e^{2}}{4 \pi \varepsilon_{0} r}$, therefore, it was naturally to wait that Eq. (1.2) will "give out" this energy as a result of the solution under the definite conditions. Indeed, the formal cutting off leads to the discrete series of values of the total electron energy

$$
\begin{equation*}
W=-\frac{Z e^{2}}{8 \pi \varepsilon_{0} a_{0}(\kappa+l+1)^{2}}=-\frac{Z e^{2}}{8 \pi \varepsilon_{0} a n^{2}} \tag{1.5}
\end{equation*}
$$

where the sum $n=\kappa+l+1$ has the name of the main quantum number and equal to $1,2,3, \ldots$ The formula (1.5) gives birth to an illusion of the solution of the problem. Actually, in a strictly scientific sense, here we deal with the plain mathematical adjustment to Bohr postulates. The power two (as in the case of $n^{2}$ ) is a usual result for the many second order equations.

In the case of H -atom ( $Z=1$ ), the expression for energy (1.5) is simplified

$$
\begin{equation*}
W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0}(\kappa+l+1)^{2}}=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} n^{2}} \tag{1.5a}
\end{equation*}
$$

The radial solution (1.3) for the hydrogen-like atom takes the form

$$
\begin{equation*}
R_{n l}=N_{n l} e^{-\rho / n}\left(\frac{2 \rho}{n}\right)^{l} L_{n-l-1}^{2 l+1}\left(\frac{2 \rho}{n}\right) \tag{1.6}
\end{equation*}
$$

where

$$
\begin{equation*}
N_{n l}=\frac{2}{n^{2}} \sqrt{\frac{(n-l-1)!}{(n+l)!}} \cdot\left(\frac{1}{a}\right)^{3 / 2} \tag{1.7}
\end{equation*}
$$

and

$$
\begin{equation*}
L_{n-l-1}^{2 l+1}\left(\frac{2 \rho}{n}\right)=\frac{1}{(n-l-1)!} e^{2 \rho / n}\left(\frac{2 \rho}{n}\right)^{-(2 l+1)} \frac{d^{n-l-1}}{d\left(\frac{2 \rho}{n}\right)^{n-l-1}}\left\{e^{-2 \rho / n}\left(\frac{2 \rho}{n}\right)^{l+n}\right\} \tag{1.8}
\end{equation*}
$$

is the standard Laguerre polynomial.

With use of the accepted designations, Schrödinger's wave function (1.1a) is represented as

$$
\begin{equation*}
\Psi=R_{n l}(r) \Theta_{l m}(\theta) \Phi_{m}(\varphi) T(t)=\psi_{n l}(r, \theta, \varphi) T(t), \tag{1.9}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi_{n l}(r, \theta, \varphi)=R_{n l}(r) \Theta_{l m}(\theta) \Phi_{m}(\varphi) \tag{1.9a}
\end{equation*}
$$

Schrödinger's radial equation (1.2) contains only the wave number $l$. The number $\kappa$ has an auxiliary character. Accordingly, the radial function should be presented in the form $R_{l}(\rho, \kappa)$. Taking into consideration these remarks, Schrödinger's elementary $\Psi$-function (1.9) can be rewritten as

$$
\begin{equation*}
\Psi_{l m, \kappa}=\psi_{l m, \kappa}(\rho, \theta, \varphi ; \kappa) T(t)=R_{l}(\rho ; \kappa) \Theta_{l m}(\theta) \Phi_{m}(\varphi) T(t), \tag{1.9b}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi_{l m, \kappa}(\rho, \theta, \varphi ; \kappa)=R_{l}(\rho ; \kappa) \Theta_{l m}(\theta) \Phi_{m}(\varphi) \tag{1.9c}
\end{equation*}
$$

is the spatial $\psi_{l m, \kappa}$-function.
According to the quantum mechanics conception, extremes of radial functions $R_{n l}$, define the radii of shells of the most probable states: $r=a \rho_{n l, \max i}$, where $i$ is the number of the root of the extremum. However, for the overwhelming number of cases, these roots are not equal to the integers squared, i.e., $\rho_{n l, \text { max } i} \neq m^{2}$ (where $m=1,2,3, \ldots$ ), and hence, they deny the cutting off condition (1.5). Such roots define the energetic levels, nonexistent in nature:

$$
\begin{equation*}
W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a \rho_{n l, \max i}} \tag{1.10}
\end{equation*}
$$

Thus, according to the condition (1.5), the energetic levels (states) (1.10) must not exist. If we suppose that they exist, then these levels must formally transform the radial function into a divergent functional series because Eq. (1.10), where $\rho \neq m^{2}(m=1,2,3, \ldots)$, does not satisfy the cutting off condition (1.5), where $\rho=n^{2}(n=1,2,3, \ldots)$. Such an absurdity appeared because of the cutting off, artificial and invalid, of the power series.

The wave numbers of Schrödinger's equation are usually compared with the quantum numbers in Bohr-Sommerfeld's generalized theory of a hydrogen atom. Between 1913 and 1926, the Bohr-Sommerfeld's theory took roots in minds; as a result, the superficial resemblance of its quantum numbers was groundlessly used by founders of quantum mechanics.

As an analog of the azimuth number $m$, the magnetic number $m$ of BohrSommerfeld's theory was accepted. The number $l$ plays the role of the azimuth number $n_{\varphi}$, which defines (along with the main quantum number $n$ ) the smaller half-axis of the elliptical electron orbit $b=a_{0} n n_{\varphi}$. The larger half-axis of the orbit $a$, defining the electron's total energy on the orbit, in Bohr's-Sommerfeld's
theory, depends only on the main quantum number $n$ : $a=a_{0} n^{2}$. Such a formal juxtaposition must mean that the wave function in Schrödinger's equation (1.1) contains the elliptical orbits in the form of "electron clouds". All these definitions are a fruit of fantasy. In fact, Schrödinger's equation describes only the circular electron orbits, but not mystic clouds-orbitals. We will make sure of it below.

Creation of any theory requires, at the initial stage, very important preliminary research work. This work, as a rule, is never published and remains forever unknown. Moreover, it can contain errors, which are hidden in the final variant of the theory. In publications, only the results of the work are presented, which, too often, are far from the initial sketches of the theory, both in the form and contents.

Let us remember the birth of Maxwell's electromagnetic theory. His contemporaries noted that Maxwell constructed his theory not following strict logic. Maxwell can plainly turn down a member, replace an unsuitable sign by the opposite one in an expression, substitute a meaning of a letter, etc. H. Poincaré noted, in connection with this, that since Maxwell's arguments and calculations contain difficult to correct errors, all six of Maxwell's equations should be accepted as an initial hypothesis, as postulates upon which we must rest in electromagnetic theory.

Schrödinger acted in approximately the same way. And we can assume that, at the initial stage of his work, of course, he cannot do without the use of the ordinary wave equation (for an arbitrary periodic process, running in space and time):

$$
\begin{equation*}
\Delta \Psi=\frac{1}{v_{0}^{2}} \frac{\partial^{2} \Psi}{\partial t^{2}} . \tag{1.11}
\end{equation*}
$$

Presenting the $\Psi$-function in the form $\Psi=\psi(x, y, z) e^{i o t}$, where $\psi(x, y, z)$ is its amplitude (a complex magnitude, in a general case), we obtain

$$
\Delta \Psi=\frac{1}{v_{0}^{2}} \frac{\partial^{2} \Psi}{\partial t^{2}}=-\frac{\omega^{2}}{v_{0}^{2}} \Psi=-k^{2} \Psi .
$$

Hence, the wave equation (1.11) can be presented as

$$
\begin{equation*}
\Delta \Psi+k^{2} \Psi=0, \tag{1.12}
\end{equation*}
$$

where $k=\frac{\omega}{v_{0}}=\frac{2 \pi}{\lambda}=\frac{1}{\lambda}$ is the wave number of the field.
Comparing equations (1.12) and (1.1), we find that the wave number $k$ in Schrödinger's equation is:

$$
\begin{equation*}
k= \pm \sqrt{\frac{2 m}{\hbar^{2}}\left(W+\frac{e^{2}}{4 \pi \varepsilon_{0} r}\right)} . \tag{1.13}
\end{equation*}
$$

It means that the wave number (entered in Schrödinger's radial equation) is a continuously variable, in the radial direction, quantity. Is it possible to image a field, where the wave number and, accordingly, the frequency change from one point to another in the space of the field? Of course, it is not possible. Such wave objects do not exist in nature!

The wave number $k$ is a constant parameter of wave objects. It can take a definite series of discrete values only in dependence on the boundary conditions.

According to the cutting off condition (1.5), the wave number (1.13) is defined by the following formula

$$
\begin{equation*}
k=e \sqrt{\frac{Z m}{2 \pi \varepsilon_{0} \hbar^{2}}\left(\frac{1}{r}-\frac{1}{2 a n^{2}}\right)} . \tag{1.13a}
\end{equation*}
$$

From this it follows that the wave number $k$ is a real number only under the condition $r<2 a n^{2}$. Therefore, one should mention the limiting sphere of wave processes in the atom. The radius of the sphere is equal to the doubled radius of $n$-th Bohr orbit (orbital):

$$
\begin{equation*}
r_{\max }=2 r_{n}=2 a n^{2} . \tag{1.14}
\end{equation*}
$$

In a case when the wave number $k$ also takes the imaginary values, the field will not be a wave field and hydrogen-like atoms will be surrounded, beyond their spheres of the radius $r_{\text {max }}$, with the field of the aperiodic structure. However, this is completely contradictory to reality. Thus, the limiting sphere bounds around the Schrödinger atom. Beyond the sphere, it is impossible to speak about the structure and wave properties of the atom.

The normalizing factors of radial functions have a conditional character, because they are determined by the integrals with an upper limit of integration, equal to infinity but not to the limiting radius (1.14). These remarks are valid also for formulae of mean values. In particular, a mean value of inverse distances, defined by the integral

$$
\begin{equation*}
\left\langle\frac{1}{r}\right\rangle=\int_{0}^{\infty} R_{n l}^{2} r d r=\left(\frac{Z}{a_{0}}\right) \frac{1}{n^{2}}, \tag{1.15}
\end{equation*}
$$

has a conditional character. Moreover, this expression is incorrect. As a matter of fact, the radial functions $R_{n l}$ define the shells of the most probable values of atomic distances in accordance with the quantum mechanical interpretation of the wave function. These distances form a discrete series, which cannot be averaged, as it is impossible to average an inverse series of distances. Indeed, suppose we need to know the mean wavelength of a hydrogen atom spectrum, for example, Balmer series. Of course, we can calculate it, but it is a senseless operation, because such a mean wave does not exist in nature.

In spite of all fittings, the mean radius of an electron orbit,

$$
\begin{equation*}
\langle r\rangle=\int_{0}^{\infty} R_{n, l}^{2} r^{3} d r=a \frac{3 n^{2}-l(l+1)}{2}, \tag{1.15a}
\end{equation*}
$$

is not proportional to $n^{2}$. Moreover, the radial spheres define the orbits of the most probable states; therefore, the radii of stationary electron orbits are constant within the corresponding spheres. Thus, the averaging (1.15a) has no sense. Further, even if we assume that the electron's motion can be the elliptic one then such orbits must pierce the shells of the stationary states. Accordingly, when an electron recedes from $H$-atom, moving along a stationary elliptic orbit, it must absorb energy, at the transition from one shell to another, and, at the approaching to $H$-atom, it must emit energy of the same value. The energetic transitions within the orbit will be determined by irrational numbers that is not observed in reality. Apart from this, such strange orbits cannot be regarded as stationary.

Let us turn now to Schrödinger's initial report, where the equation of quantum mechanics (1.1) was first presented on the basis of the operator and variational methods. We will consider this in an elementary form.

Any material object is characterized by the kinetic and potential energies which define its total energy

$$
\begin{equation*}
E=\frac{p^{2}}{2 m}+U . \tag{1.16}
\end{equation*}
$$

We introduce the $\Psi$-function of zero dimensionality, complex in a general case; its field gradient is the momentum of a microparticle, defined by the equation

$$
\begin{equation*}
\mathbf{p}=i \hbar\left(\frac{\partial \Psi}{\partial x} \mathbf{e}_{x}+\frac{\partial \Psi}{\partial y} \mathbf{e}_{y}+\frac{\partial \Psi}{\partial z} \mathbf{e}_{z}\right)=i \hbar \nabla \Psi, \tag{1.17}
\end{equation*}
$$

where $\hbar$ is some elementary action, needed for realization of the law of equality of dimensionalities of the left and right parts in Eq. (1.17); $i$ is the imaginary unit. Inasmuch as, in a general case, the $\Psi$-function is a complex one, components of the momentum are also, in a general case, complex ones. However, their real parts (by definition) represent the ordinary projections of the momentum along the coordinate axes. Thus, the real part of the complex momentum defines the momentum of a microparticle:

$$
\begin{equation*}
\mathbf{p}=\operatorname{Re}(i \hbar \nabla \Psi) \tag{1.18}
\end{equation*}
$$

Relying on the expression (1.17), we can represent the energy (1.16) in the following way

$$
E=-\frac{\hbar^{2}}{2 m}\left(\left(\frac{\partial}{\partial x}\right)^{2}+\left(\frac{\partial}{\partial y}\right)^{2}+\left(\frac{\partial}{\partial z}\right)^{2}\right) \Psi+U,
$$

or

$$
\begin{equation*}
E=-\frac{\hbar^{2}}{2 m} \Delta \Psi+U . \tag{1.19}
\end{equation*}
$$

Let us now introduce operators of the total and potential energies, $\hat{H}$ and $\hat{U}$, according to the following expressions:

$$
\begin{equation*}
E=\hat{H} \Psi, \quad U=\hat{U} \Psi . \tag{1.20}
\end{equation*}
$$

Substituting $E$ and $U$, in Eq. (1.19), with these operator expressions, we will have

$$
\begin{equation*}
\hat{H} \Psi=-\frac{\hbar^{2}}{2 m} \Delta \Psi+\hat{U} \Psi . \tag{1.21}
\end{equation*}
$$

or

$$
\begin{equation*}
\Delta \Psi+\frac{2 m}{\hbar^{2}}(\hat{H}-\hat{U}) \Psi=0 . \tag{1.21a}
\end{equation*}
$$

In a case with hydrogen-like atoms, we seek the field of such a $\Psi$-function for which the following equalities must exist:

$$
\begin{equation*}
\hat{H}=W, \quad \hat{U}=-\frac{Z e^{2}}{4 \pi \varepsilon_{0} r} \tag{1.22}
\end{equation*}
$$

As a result, if we accept $Z=1$, we arrive (as Schrödinger assumed) at the wave equation for the electron in the $H$-atom

$$
\begin{equation*}
\Delta \Psi+\frac{2 m}{\hbar^{2}}\left(W+\frac{Z e^{2}}{4 \pi \varepsilon_{0} r}\right) \Psi=0 . \tag{1.23}
\end{equation*}
$$

Taking into consideration the expression (1.13), we deduce Eq. (1.23) to the standard form:

$$
\begin{equation*}
\Delta \Psi+k^{2} \Psi=0 . \tag{1.23a}
\end{equation*}
$$

The following question arises: why did Schrödinger not use the standard wave equation? There are several answers to this question. They reflect the different aspects of a situation, fully developed in physics in the 1920's.

Before the appearance of Schrödinger's works, the quantum ideology has already exerted its influence on the minds of physicists. Quanta, as the elements of discreteness, as the atoms of properties, had been considered separately from the wave nature of matter-space-time. And this situation still remains, in physics, up to this day.

In fact, nature is the symmetrical structure of contradictions inseparable from each other. This also concerns such a contradictory pair as discontinuitycontinuity. Wave fields represent continuity; periods-measures of properties of wave objects and fields represent its opposite side - discontinuity. These peri-ods-measures are the physical manifestation of what is accepted as the discrete nature of matter.

In wave processes, transitions from one dynamically stable state to another usually occur through jumps. These jumps by themselves represent transient
processes, occurring during a very short time, which physics, all too often, cannot detect. A specific complicated frequency structure is inherent for a jump.

For example, if we would attempt to hear the constant sound of a cross-cut saw (or a plane metal plate), beginning from a low frequency and progressing to higher frequencies, while fluently bending the saw, we will find that frequency will not have changed continuously. Frequency can suddenly increase, by jump, up to a relatively high value. We can also find such an unstable state when an insignificant change of the pressure on a fiddlestick changes the note *.

Quantum formalism has interpreted quantum jumps (in accordance with Einstein's views) as transitions from one energy level to another without intermediate states, i.e., as processes occurring out of space and time. That is, they are equivalent to transitions with infinite speed. Einstein suggested such a primitive model. His energy quantum arises instantly with the wave speed of motion. And H. Weil, in the spirit of the age, "helped" Schrödinger (by means of the formal cutting off of the infinite solutions) to obtain a theoretical justification for such a myth. However, Schrödinger had understood all the conditionality of this solution, and he had rightly called the quantum jumps, out of space and time, the "cursed" jumps.

As known, any wave equation is the equation of mass processes. It describes the result of the interaction of particles and subparticles in space, from which the waves arise. Wave mass processes represent the kinematic level of motion, or the level of superstructure, below which is the level of interaction, or the level of basis.

Because of this, Schrödinger's equation is unable to describe the motion of the only electron. In spite of this, at that time, physicists groundlessly ascribed to Schrödinger's equation a nonexistent aptitude (unnatural for wave equations in principle). They assumed it must describe the motion of the single electron in the hydrogen atom. This was a gross blunder.

An introduction, in kinematic wave equations, of potentials or potential energies of an interaction means a lack of understanding of discriminate differences between the dynamic basis of wave, i.e., the level of mass coordinated interaction, and the level of superstructure of wave, i.e., the ordered kinematic motion.

The divergence of the power series of the radial function in Schrödinger's equation is the effect of mixing the kinematic and dynamic levels of motion, which were formally (incorrectly in essence) joined together in Schrödinger's equation.

Such an approach exerted significant influence on the development of microworld theories. It gave birth to the phenomenological constructions and, correspondingly, definite scientific schools. The latter are represented through numerous scientific journals, which take away science from the real picture of

[^0]the microworld. Things reached a crisis point; therefore, it is necessary to analyze in detail the errors of quantum mechanics, and, most of all, the logically and physically erroneous interpretation of complex $\Psi$-functions.

For this purpose, we again return to operator formalism, which was created by "lazy" theorists, since operator methods are simpler than functional calculations.

Let us present Eq. (1.23a) in the form of the product of the $\Psi$-function by the operator binomial as

$$
\begin{equation*}
\left(\Delta+k^{2}\right) \Psi=0 \tag{1.24}
\end{equation*}
$$

Because the $\Psi$-function is unequal to zero, we obtain the operator equation for the simplest value of the operator $\Delta$ (regarding $\Delta$ as a variable operator magnitude): $\Delta+k^{2}=0$. Solving this quadratic equation, we have

$$
\begin{equation*}
\Delta=-k^{2}, \quad \text { and } \quad \nabla=-i \mathbf{k}, \tag{1.25}
\end{equation*}
$$

since $\Delta=\nabla^{2}$. In the wave $\Psi$-field, the momentum (1.17) of an arbitrary particle takes the form

$$
\begin{equation*}
\mathbf{p}=i \hbar \nabla \Psi=\mathbf{k} \hbar \Psi, \tag{1.26}
\end{equation*}
$$

or, in the scalar form,

$$
\begin{equation*}
p=i \hbar \nabla \Psi=k \hbar \Psi . \tag{1.27}
\end{equation*}
$$

What does this equality represent by itself?
Any physical parameter $P$ (of an arbitrary physical wave field) has its own fundamental wave measure, or a period-quantum $P_{q}$. Using this quantum, the value of a parameter $P$ can be presented by the quantitative relative $\Psi$-measure:

$$
\begin{equation*}
\Psi=\frac{P}{P_{q}} . \tag{1.28}
\end{equation*}
$$

In a general case, the parameter $P$ is the complex quantity

$$
\begin{equation*}
P=p_{k}+i p_{p} . \tag{1.29}
\end{equation*}
$$

Let us agree to call the "real" part of Eq. (1.29) the "kinetic" component and the "imaginary" part, the "potential" component of $P$-parameter. (The usefulness of this terminology will be demonstrated later on)

By virtue of this, $\Psi$-measure of the zero physical dimensionality will be a complex wave function with the argument,

$$
\begin{equation*}
\arg =i(\omega t-\mathbf{k r})=i\left(\omega t-k_{x} x-k_{y} y-k_{z} z\right), \tag{1.30}
\end{equation*}
$$

which indicates that the quantitative measure of $P$-parameter is changed in space and time. The presence, in (1.30), of the imaginary unit $i$ is not casual. It simplifies calculations and has a deep philosophical sense, which will be revealed in the following chapters.

The argument (1.30) meets general physical principles. Thus, the wave structure of any physical parameter $P$ is presented by the following scalar measure:

$$
\begin{equation*}
P=P_{q} \Psi\left(i\left(\omega t-k_{x} x-k_{y} y-k_{z} z\right)\right) . \tag{1.31}
\end{equation*}
$$

If $P$ is the momentum, then Eq. (1.27) can be written as

$$
\begin{equation*}
p=k \hbar \Psi\left(i\left(\omega t-k_{x} x-k_{y} y-k_{z} z\right)\right) \tag{1.32}
\end{equation*}
$$

The relative elementary harmonic measure

$$
\begin{equation*}
\Psi=\Psi_{m} \exp \left(i\left(\omega t-k_{x} x-k_{y} y-k_{z} z\right)\right) \tag{1.32a}
\end{equation*}
$$

of any parameter $P$ satisfies the differential equations with: (1) the spatial partial derivatives of the second order

$$
\begin{equation*}
\frac{\partial^{2} \Psi}{\partial x^{2}}=-k_{x}^{2} \Psi, \quad \frac{\partial^{2} \Psi}{\partial y^{2}}=-k_{y}^{2} \Psi, \quad \frac{\partial^{2} \Psi}{\partial z^{2}}=-k_{z}^{2} \Psi \tag{1.33}
\end{equation*}
$$

or

$$
\begin{equation*}
\Delta \Psi=-\left(k_{x}^{2}+k_{y}^{2}+k_{z}^{2}\right) \Psi=-k^{2} \Psi, \tag{1.33a}
\end{equation*}
$$

and (2) the partial time derivative of the second order

$$
\begin{equation*}
\frac{\partial^{2} \Psi}{\partial t^{2}}=-\omega^{2} \Psi \tag{1.33b}
\end{equation*}
$$

Equations (1.33a) and (1.33b) form the wave equation of the harmonic $\Psi$ function:

$$
\begin{equation*}
\Delta \Psi=\frac{1}{v_{0}^{2}} \frac{\partial^{2} \Psi}{\partial t^{2}} \tag{1.34}
\end{equation*}
$$

Obviously, the sum of elementary measures constitutes the measure of the general character; therefore, we assume that Eq. (1.34) also defines the wave field of the measure of an arbitrary parameter. Because in any point under the steady-state wave motion the product of its spatial (amplitude) $\psi(\mathbf{k r})$ and time $\hat{T}(\omega t)$ components represents $\Psi$-function, the wave equation (1.34) therefore falls into the amplitude and time equations:

$$
\begin{equation*}
\Delta \psi+k^{2} \psi=0, \quad \frac{\partial^{2} \hat{T}}{\partial t^{2}}=-\omega^{2} \hat{T} . \tag{1.34a}
\end{equation*}
$$

The constant parameters, $k$ and $\omega$, are determined on the basis of boundary conditions. But if these parameters are difficult to determine, then we have to remember that nature itself solves this problem. We need only find, empirically, a series of the discrete values of $k$ and $\omega$ and try to understand it.

Since equations (1.34) and (1.34a) describe $\Psi$-measures of arbitrary physical parameters, the difference of their wave structure comes down to the difference of kinematic types of the corresponding wave fields. The basic wave fields are the plane, cylindrical, spherical, and complicated (spherical-cylindrical) fields. Therefore, these fields, to an equal degree, successfully describe not only the atomic structure, but also the structure of megaobjects.

## 2. $\Psi-F u n c t i o n s$ of the wave field of a string

First, for the sake of simplicity and clarity, we will consider the logic of errors by quantum mechanics, analyzing oscillations of a homogeneous string. We are interested in small transversal oscillations of the string near the equilibrium (Fig. 3.1).


Fig. 3.1. On the problem of oscillations of a string with the length $l$, fixed at both ends; $S$ is a cross-section of oscillations, $m$ is the mass of an arbitrary part of the string in the cross-section $S$.

Every point of the string is defined by the coordinate $z$, the state of its motion in time $t$-by the displacement from the equilibrium, $x(z, t)$, and the motion itself - by the complex $\Psi$-function

$$
\begin{equation*}
\Psi(z, t)=x(z, t)+i y(z, t) \tag{2.1}
\end{equation*}
$$

Displacements of points of the string from the equilibrium are equal to the real part of $\Psi$-function:

$$
\begin{equation*}
x(z, t)=\operatorname{Re} \Psi(z, t) \tag{2.2}
\end{equation*}
$$

For the description of oscillations of the string, we use Euler's formula $e^{i \varphi}=\cos \varphi+i \sin \varphi$ with its famous imaginary unit $i$. In order to get rid of this unit, creators of quantum mechanics proposed the well-known interpretation of the wave $\Psi$-function for $H$-atom. On the occasion of this interpretation, an epigram devoted to Schrödinger was spread among physicists. The epigram, composed in English and German languages, ascribed to E. Hückel (1896-1980, German physicist-theorist), is as follows:

Erwin with his psi can do
Calculations quite a few But one thing has not been seen Just what does psi really mean?

> Gar Machens rechnet Erwin schon
> Mit seiner Wellenfunktion
> Nur wissen möcht'man gerne wohl
> Was man sich dabei vorstell'n soll?

This epigram rightly comments on the real situation concerning the interpretation of the "psi" function. And what is more, a careful analysis brings to light the fact that Schrödinger's calculations are not in conformity with experimental data, which these functions (as is usually accepted) must describe. And we will demonstrate this below.

In the case of the wave field of the string, $\Psi$-function satisfies the wave equation for the string

$$
\begin{equation*}
\Delta \Psi=\frac{1}{v_{0}^{2}} \frac{\partial^{2} \Psi}{\partial t^{2}} \tag{2.3}
\end{equation*}
$$

Here, $\Delta=\frac{\partial^{2}}{\partial z^{2}}$ is the one-dimensional Laplacian operator; $v_{0}$ is the wave speed in the string defined by the following expressions

$$
\begin{equation*}
v_{0}=\sqrt{\frac{T_{S} l}{M}}, \quad T_{S}=E \frac{\Delta l}{l} S, \tag{2.4}
\end{equation*}
$$

where $T_{S}$ is the tension, $l$ is the length, $M$ is the mass, $E$ is Young modulus, $\Delta l$ is the lengthening, and $S$ is the area of the cross-section of the string.

Let us agree to call the "real" component of the complex displacement $\operatorname{Re} \Psi(z, t)=x(z, t)$ the potential displacement and the "imaginary" component $\operatorname{Im} \Psi(z, t)=y(z, t)$, the kinetic displacement.

The conjugated displacements make it possible to more completely describe the wave field of the string, as the potential-kinetic wave field. An introduction of these notions entirely complies with the demands of the laws of dialectics, i.e., with the philosophical and logical system treating the Universe as a symmetrical system of oppositions.

We will seek an elementary solution of the wave equation in the form of the product of the string's space displacements, represented by the function $\psi(k z)$, and the space of time of wave events, expressed by the function $\hat{T}(\omega t)$ :

$$
\begin{equation*}
\Psi(z, t)=\psi(k z) \hat{T}(\omega t), \tag{2.5}
\end{equation*}
$$

where $k=\frac{\omega}{v_{0}}=\frac{2 \pi}{\lambda}$ is the wave number, $\omega$ is the circular frequency of oscillations, and the sign ^ over the symbol of the time function $\hat{T}(\omega t)$ does not allow confusion with the period $T$.

The function (2.5) is the mathematical expression of the indissoluble bond of material and time spaces, or (rather) the fields of material space and physical time, which are the major oppositions in the Universe.

The function $\hat{T}(\omega t)$ expresses the alternating physical time field by means of the argument $t$, which represents the ideal mathematical time of the imaginary absolute uniform motion, which is nonexistent in nature. Timers of different
types realize this imaginary time with an approximation. Such a preliminary explanation of $\hat{T}$-function, as an image of the alternating physical field of time, is quite abstract; therefore, further, this notion will be supplemented with concrete content.

The existence of two spaces-fields (2.5) allows representation of the wave equation of the superstructure (2.3) through equations of the space and time of the superstructure:

$$
\begin{align*}
& \Delta \psi+k^{2} \psi=0,  \tag{2.6}\\
& \frac{\partial^{2} \hat{T}}{\partial t^{2}}=-\omega^{2} \hat{T} . \tag{2.6a}
\end{align*}
$$

These equations define two elementary plane-polarized transversal waves, travelling towards each other:

$$
\begin{align*}
& \Psi^{+}=A e^{i\left(\varphi_{0}-k z\right)} e^{i \omega t}=A e^{i\left(\omega t-k z+\varphi_{0}\right)}=A\left(\cos \left(\omega t-k z+\varphi_{0}\right)+i \sin \left(\omega t-k z+\varphi_{0}\right)\right), \\
& \Psi^{-}=A e^{-i\left(\varphi_{0}-k z\right)} e^{i \omega t}=A e^{i\left(\omega t+k z-\varphi_{0}\right)}=A\left(\cos \left(\omega t+k z-\varphi_{0}\right)+i \sin \left(\omega t+k z-\varphi_{0}\right)\right) . \tag{2.7}
\end{align*}
$$

Upper indexes of $\Psi$-functions indicate the direction of waves motion: the sign " + " designates motion in the positive direction and the sign "-", in the negative direction along the Z -axis with the speed $v_{0} ; A$ is the amplitude of waves, which, generally speaking, can take any complex value. In a particular case, under the name the amplitude, we will mean any real number.

Traveling waves $\Psi^{+}$and $\Psi^{-}$define the potential and kinetic displacements in the following forms:

$$
\begin{array}{ll}
x^{+}=\operatorname{Re} \Psi^{+}=A \cos \left(\omega t-k z+\varphi_{0}\right), & y^{+}=\operatorname{Im} \Psi^{+}=A \sin \left(\omega t-k z+\varphi_{0}\right) . \\
x^{-}=\operatorname{Re} \Psi^{-}=A \cos \left(\omega t+k z-\varphi_{0}\right), & y^{-}=\operatorname{Im} \Psi^{-}=A \sin \left(\omega t+k z-\varphi_{0}\right) . \tag{2.9}
\end{array}
$$

The addition of the two traveling waves results in the standing wave:

$$
\begin{equation*}
\Psi=\Psi^{+}+\Psi^{-}=a \cos \left(k z-\varphi_{0}\right) e^{i \omega t} . \tag{2.10}
\end{equation*}
$$

It defines the potential and kinetic displacements:

$$
\begin{align*}
& x=x^{+}+x^{-}=\operatorname{Re} \Psi=a \cos \left(k z-\varphi_{0}\right) \cos \omega t  \tag{2.10a}\\
& y=y^{+}+y^{-}=\operatorname{Im} \Psi=a \cos \left(k z-\varphi_{0}\right) \sin \omega t \tag{2.10b}
\end{align*}
$$

where $a=2 A$ is the amplitude of the standing wave.
The boundary condition for the string, fixed on the ends, is

$$
\begin{equation*}
\operatorname{Re} \Psi(k z, t)=\left.a \cos \left(k z-\varphi_{0}\right)\right|_{z=0}=a \cos \left(k z-\varphi_{0}\right)_{z=l}=0 . \tag{2.11}
\end{equation*}
$$

This condition is realized if

$$
\varphi_{0}=\pi / 2 \quad \text { and } \quad k=\frac{\omega}{v_{0}}=\frac{2 \pi}{\lambda}=\frac{n \pi}{l}, \quad \text { or }
$$

$$
\begin{equation*}
\omega=\frac{2 \pi v_{0}}{\lambda}=\frac{n \pi}{l} v_{0}, \quad \text { or } \quad l=\frac{n}{2} \lambda=v \lambda \tag{2.12}
\end{equation*}
$$

where $n=1,2,3, \ldots$ is the number of half-waves and $v=\frac{n}{2}$ is the number of waves placed on the string.

The condition $n=1$ defines the parameters of the fundamental tone of the string:

$$
\begin{equation*}
k_{1}=\frac{\omega_{1}}{v_{0}}=\frac{2 \pi}{\lambda_{1}}=\frac{\pi}{l}, \quad \omega_{1}=\frac{2 \pi v_{0}}{\lambda_{1}}=\frac{\pi}{l} v_{0} \quad \text { and } \quad l=\frac{1}{2} \lambda_{1} . \tag{2.13}
\end{equation*}
$$

Under the realization of the conditions presented above, $\Psi$-function describes the following elementary traveling waves of the space-time:

$$
\begin{align*}
& \Psi^{+}=A e^{i(\omega t-k z+\pi / 2)}=A(-\sin (\omega t-k z)+i \cos (\omega t-k z))  \tag{2.14}\\
& \Psi^{-}=A e^{i(\omega t+k z-\pi / 2)}=A(\sin (\omega t+k z)-i \cos (\omega t+k z)) \tag{2.14a}
\end{align*}
$$

with the potential and kinetic displacements

$$
\begin{array}{ll}
x^{+}=\operatorname{Re} \Psi^{+}=-A \sin (\omega t-k z), & y^{+}=\operatorname{Im} \Psi^{+}=A \cos (\omega t-k z) \\
x^{-}=\operatorname{Re} \Psi^{-}=A \sin (\omega t+k z), & y^{-}=\operatorname{Im} \Psi^{-}=-A \cos (\omega t+k z) \tag{2.15a}
\end{array}
$$

Following the solutions obtained, the potential and kinetic displacements of traveling (in space and time) waves have the phase lag by a quarter of the period, $90^{\circ}$. Let us agree on the graphs of the potential and kinetic displacements to present with the two mutually perpendicular axes, $x^{+}\left(x^{-}\right)$and $y^{+}\left(y^{-}\right)$(Fig. 3.2).



Fig. 3.2. Traveling (in space and time) potential-kinetic waves $\Psi^{+}$and $\Psi^{-}$at the instant $t=0$; white circles define points of extremes of the kinetic displacements and dark circles - points of extremes of the potential displacements.
The traveling waves form the standing wave (Fig. 3.3):

$$
\begin{equation*}
\Psi=\Psi^{+}+\Psi^{-}=a \sin k z e^{i \omega t}=x+i y \tag{2.16}
\end{equation*}
$$

In the Eq. (2.16),

$$
\begin{equation*}
x=\operatorname{Re} \Psi=a \sin k z \cos \omega t, \quad y=\operatorname{Im} \Psi=a \sin k z \sin \omega t \tag{2.16a}
\end{equation*}
$$

or, briefly,

$$
\begin{equation*}
x=\operatorname{Re} \Psi=a_{z} \cos \omega t, \quad y=\operatorname{Im} \Psi=a_{z} \sin \omega t \tag{2.16b}
\end{equation*}
$$

where $a_{z}=a \sin k z$ is the amplitude of oscillations of the standing wave in the point $z$. It defines the points of the string, in which potential displacements are maximal. We call such points the potential points of the wave field of the string.

a)

b)

c)

Fig. 3.3. (a) A graph of the potential-kinetic standing $\Psi$-wave of the field of the string at the instant $t=0$ (signs " + " and "-" show the domains of opposite phases of oscillations, $\pm a_{z}= \pm a \sin k z$ which are amplitudes of oscillations of opposite phases in a cross-section $Q$ ); (b) a graph of the potential-kinetic wave (in space and time) with the potential and kinetic displacements, $x$ and $y$, at the instant $t \neq n \pi$, where $n$ is an integer (displacements $x$ and $y$, in space, coincide in phase and are shifted, in time, by a quarter of the period, $90^{\circ}$; (c) a graph of the potential-kinetic oscillations in time in the standing wave, in an arbitrary point $z$, satisfying the condition $k z \neq n \pi$.

On $Z$-axis, points of the string are characterized by maximal kinetic displacements; therefore, we call Z-axis the axis of kinetic points of the wave field of the string.

On the basis of (2.16a), we find the kinetic speed equal to the first derivative of the potential displacement with respect to time:

$$
\begin{equation*}
v_{k}=\frac{d x}{d t}=\frac{\operatorname{Re} \Psi}{d t}=-\omega a \sin k z \sin \omega t=-\omega y, \tag{2.17}
\end{equation*}
$$

which is proportional to the kinetic displacement y.
By analogy to the definition of the kinetic speed, we call the derivative of the kinetic displacement with respect to time the potential speed:

$$
\begin{equation*}
v_{p}=\frac{d y}{d t}=\frac{d \operatorname{Im} \Psi}{d t}=\omega a \sin k z \cos \omega t=\omega x \tag{2.17a}
\end{equation*}
$$

which is proportional to the potential displacement $x$.
Kinetic and potential speeds form the kinetic-potential complex speed

$$
\begin{equation*}
v=\frac{d \Psi}{d t}=v_{k}+i v_{p}=-\omega y+i \omega x \tag{2.18}
\end{equation*}
$$

which defines the kinetic-potential momentum of the material object:

$$
\begin{equation*}
p=m v=m \frac{d \Psi}{d t}=m v_{k}+i m v_{p}=-m \omega y+i m \omega x \tag{2.19}
\end{equation*}
$$

where

$$
\begin{equation*}
p_{k}=m v_{k}=-m \omega y, \tag{2.19a}
\end{equation*}
$$

$$
\begin{equation*}
p_{p}=m v_{p}=m \omega x \tag{2.19b}
\end{equation*}
$$

are the kinetic and potential momenta, correspondingly.
As we see, the kinetic displacement defines the kinetic momentum. In turn, the potential displacement defines the potential momentum. The kinetic displacement and speed, $y$ and $v_{k}$, correspond to the conventional classical notions of the displacement and speed.

Let us clarify the meaning of the potential displacement and speed.
Any atom of the string, as a physical point of its space of the mass $m_{A}$ (Fig. 3.1), in the wave field of the standing wave is characterized by the additional kinetic $\varepsilon_{k}$ and potential $\varepsilon_{p}$ energies, which are determined over the, correspondingly, kinetic and potential parameters:

$$
\begin{align*}
& \varepsilon_{k}=\frac{m_{A} v_{k}^{2}}{2}=\frac{p_{k}^{2}}{2 m_{A}}=\frac{\mu y^{2}}{2}=\frac{m_{A} v_{m}^{2}}{2} \sin ^{2} k z \cdot \sin ^{2} \omega t,  \tag{2.20}\\
& \varepsilon_{p}=\frac{m_{A} v_{p}^{2}}{2}=\frac{p_{p}^{2}}{2 m_{A}}=\frac{\mu x^{2}}{2}=\frac{m_{A} v_{m}^{2}}{2} \sin ^{2} k z \cdot \cos ^{2} \omega t \tag{2.20a}
\end{align*}
$$

where $\mu=m_{A} \omega^{2}$ is the elasticity coefficient and $v_{m}=\omega a$ is the limiting speed of oscillations. These expressions represent the standing waves of energy. The symmetric representation of energies points out the definite symmetry of restmotion.

The definition of the potential energy (2.20a) is the classical definition; it has arithmetic character. Whilst, strictly speaking, it should be determined on the basis of the potential speed with the measure $i v_{p}$. In such a case, the potential energy will be a negative value

$$
\begin{equation*}
\varepsilon_{p}=\frac{m_{A}\left(i v_{p}\right)^{2}}{2}=-\frac{\mu x^{2}}{2}=-\frac{p_{p}^{2}}{2 m_{A}}=-\frac{m_{A} v_{m}^{2}}{2} \sin ^{2} k z \cdot \cos ^{2} \omega t \tag{2.20b}
\end{equation*}
$$

It is an algebraic definition of energy: kinetic and potential energies must have different signs, because these express the opposite states of the field - motion and rest. This is why, as soon as we turn to motion in a central field, we must operate with the negative potential energy.

The sum of potential and kinetic energies will be the variable value:

$$
\begin{equation*}
\varepsilon=\varepsilon_{k}+\varepsilon_{p}=-\frac{m_{A} v_{m}^{2}}{2} \sin ^{2} k z \cos 2 \omega t \tag{2.21}
\end{equation*}
$$

The total energy, in the wave field of a standing wave of the string, performs harmonic oscillations with double frequency. It confirms the fact that rest and motion are shifted with respect to each other, by phase, to a quarter of the period. On the other hand, the difference of the kinetic and potential energies is the constant quantity equal the amplitude of oscillations of the energies:

$$
\begin{equation*}
\varepsilon_{m}=\varepsilon_{k}-\varepsilon_{p}=\frac{m_{A} v_{m}^{2}}{2} \sin ^{2} k z \tag{2.22}
\end{equation*}
$$

Every atom is also characterized by the kinetic and potential actions, defined by measures

$$
\begin{equation*}
\hbar_{k}=p_{k} y \quad \text { and } \quad \hbar_{p}=p_{p} x \tag{2.23}
\end{equation*}
$$

and by the amplitude action

$$
\begin{equation*}
\hbar=m_{A} v_{z} a_{z}, \tag{2.23a}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{z}=a \sin k z, \quad v_{z}=a_{z} \omega=a \omega \sin k z . \tag{2.23b}
\end{equation*}
$$

The difference of symmetrical energies defines the amplitude of oscillations of energy of an atom in a cross-section $Q$ of the string with the coordinate $z$ :

$$
\begin{equation*}
\varepsilon_{m}=\varepsilon_{k}-\varepsilon_{p}=\frac{\mu a^{2} \sin ^{2} k z}{2}=\frac{\mu a_{z}^{2}}{2}=\frac{\hbar \omega}{2} . \tag{2.24}
\end{equation*}
$$

The potential speed and energy are maximal, in value, in potential points, while the kinetic ones are maximal on the Z-axis.

As the potential displacement increases in value, the potential momentum and potential energy increase; the kinetic displacement, kinetic momentum, and kinetic energy decrease.

The potential displacement and speed are measures of the potential wave field of the string. And the kinetic displacement and speed are measures of the kinetic wave field of the string. The complex displacement, speed, and momentum are potential-kinetic parameters, accurately describing the potentialkinetic wave field of the string.

It is possible to say that the kinetic speed is the measure of the intensity of motion, i.e., the kinetic field, and the potential speed is the measure of the intensity of rest, i.e., the potential field.

Potential and kinetic speeds and, corresponding to them, momenta, like potential and kinetic displacements, are also shifted in phase with respect to each other by $90^{\circ}$. This phase lag explains the unbalanced state of kinetic and potential fields as well as their mutual transformation one into another, which, under the ideal conditions of absence of energy loss, lasts eternally.

On the basis of the conjugated kinetic and potential parameters, the formulae of energies obtain a symmetrical form, reflecting the fundamental feature of all physical fields, as potential-kinetic fields (for the complete description of which the conjugated parameters are necessary).

We perceive some parameters of kinetic and potential fields visually, other ones - physically. For example, a potential displacement is perceived visually, whereas a potential momentum and a potential speed are perceived physically only over the influence upon an object of a compressed elastic system.

The potential-kinetic exchange of motion-rest in the wave kinetic-potential field is described by the potential-kinetic rate of exchange of the motion-rest
state, or briefly, by the kinema ("force") $F$, i.e., by the rate of change of poten-tial-kinetic momentum. The rate of exchange of momentum is

$$
\begin{equation*}
F=m_{A} \frac{d^{2} \Psi_{v}}{d t^{2}}=\frac{d p}{d t}=F_{p}+F_{k}=m_{A} \frac{d v_{k}}{d t}+i m_{A} \frac{d v_{p}}{d t}=-m_{A} \omega^{2}(x+i y) \tag{2.25}
\end{equation*}
$$

where

$$
\begin{equation*}
F_{p}=\frac{d p_{k}}{d t}=m_{A} \frac{d v_{k}}{d t}=-m_{A} \omega^{2} x=-\mu x \tag{2.25a}
\end{equation*}
$$

is the potential kinema, or the potential power, in a wide sense of this word (we will consider these notions in detail further, in Chapter 4), or the "potential force" of the exchange of motion, and

$$
\begin{equation*}
F_{k}=\frac{d i p_{p}}{d t}=m_{A} \frac{d i v_{p}}{d t}=-m_{A} \omega^{2} i y=-\mu i y \tag{2.25b}
\end{equation*}
$$

is the kinetic kinema, or the kinetic power of exchange of rest, or the "kinetic force".

An integral action of the potential kinema (2.25a) defines the potential energy

$$
\begin{equation*}
\varepsilon_{p}=\int_{0}^{x} F_{p} d x=-\int_{0}^{x} \mu x d x=-\frac{\mu x^{2}}{2}=\frac{m_{A}\left(i v_{p}\right)^{2}}{2} \tag{2.26}
\end{equation*}
$$

and an integral action of the kinetic kinema (2.25b) defines the kinetic energy

$$
\begin{equation*}
\varepsilon_{k}=\int_{0}^{x} F_{k} d i y=-\int_{0}^{x} \mu i y d i y=\frac{\mu y^{2}}{2}=\frac{m_{A} v_{k}^{2}}{2} \tag{2.27}
\end{equation*}
$$

Thus, the kinetic and potential energies have different signs that is in conformity with the above-accepted definitions. In some cases, when it does not matter, we will omit the minus sign of the potential energy.

It should be noted that the sign of any quantity, including kinetic and potential energies, depends on the choice of a zero-point (e.g., a zero level of energy) and the direction of reading. This concerns all physical quantities. Accordingly, none absolutism of signs can be: it is only important that the opposite measures will have the different signs and nothing more.

For example, we describe the wave field of the string by $\Psi$-function. However, $i \Psi$-function (i.e., $\Psi$-function multiplied by the "imaginary" unit) also is the solution of the wave equation. But now, the kinetic speed will be the "imaginary" quantity and the potential speed - the "real" one. As a result, the kinetic energy will be the negative quantity and the potential energy will be the positive one. And this is rightfully.

When the points of the string pass through kinetic points, the "elastic force", i.e., the potential kinema, vanishes; while the "motion force", i.e., the kinetic kinema, reaches maximal value and it is perceived, visually, as the inertia of motion.

We see, thus, that the kinema, as the physical parameter of exchange of momentum, is the broader notion than the "force" related, in a definite degree, to physiological physics. This is why, the integrals, (2.26) and (2.27), were called the integral actions of kinemas, but not the integrals of "work".

As follows from the formulae of definition of kinemas, the extremal rate of change of kinetic momentum, i.e., the potential kinema of exchange of motion $F_{p}$, takes place in potential points of the field of the string. Analogously, the extremal rate of change of potential momentum, i.e., the kinetic kinema of exchange of rest $F_{k}$, takes place on the line of kinetic points.

The above-considered potential and kinetic energies are measures of the transversal potential and kinetic wave field of the string, which relates to the superstructure of the field of the string (to the oscillatory level). These are transversal energies in the plane-polarized wave.

Now, we will supplement the transversal energies with the conjugated longitudinal energies of the basis of the wave field. The basis is related with the dynamics of transmission of the wave signal.

Generally, it is necessary for all the parameters of the superstructure of the wave to be supplemented with the conjugated parameters of the basis of the $\boldsymbol{w a v e}$. A general structure of the potential-kinetic wave can be expressed by the graph shown in Fig. 3.4).


Fig. 3.4. A graph of the potential-kinetic wave, representing its generalized model.

The potential energy of the field of basis of the string $W_{p}$ has the form

$$
\begin{equation*}
W_{p}=\frac{\mu(\Delta l)^{2}}{2}=\frac{E S(\Delta l)^{2}}{2 l} . \tag{2.28}
\end{equation*}
$$

Taking into consideration Eq. (2.4) and assuming $M=\rho S l$, after elementary transformations, we find the longitudinal potential energy per one atom of the string $\varepsilon_{p}$ :

$$
\begin{equation*}
\varepsilon_{p}=\frac{W_{p}}{M / m_{A}}=\frac{m_{A} E(\Delta l)^{2}}{2 \rho l^{2}}=\frac{m_{A} \varepsilon_{T} v_{0}^{2}}{2}, \tag{2.28a}
\end{equation*}
$$

where $\varepsilon_{T}=\frac{\Delta l}{l}$ is the relative lengthening of the string. As a result we have

$$
\begin{equation*}
\varepsilon_{p}=\frac{m_{A} \varepsilon_{T} v_{0}^{2}}{2}=\frac{m_{A} v_{0 p}^{2}}{2}, \tag{2.29}
\end{equation*}
$$

where $v_{0 p}=v_{0} \sqrt{\varepsilon_{T}}$ is the potential speed.
Since every atom takes part also in the transmission of the wave signal along the string, hence, it has the longitudinal kinetic energy of the basis of the wave, which is defined by the speed of the wave field:

$$
\begin{equation*}
\varepsilon_{k}=\frac{m_{A} v_{0}^{2}}{2} \tag{2.30}
\end{equation*}
$$

It makes sense also to speak about the wave action of an atom during the transmission of a signal along the whole length of the string, which is defined by the equality: $h=m_{A} v_{0} l$. It is the action at the level of the wave basis. Taking into account that $l=\frac{n}{2} \lambda$, we obtain

$$
\begin{equation*}
h=m_{A} v_{0} l=\frac{n}{2} h_{\lambda}, \tag{2.31}
\end{equation*}
$$

where

$$
\begin{equation*}
h_{\lambda}=m_{A} v_{0} \lambda \tag{2.31a}
\end{equation*}
$$

is the wave action. If $n=1$, we have

$$
\begin{equation*}
h=m_{A} v_{0} l=\frac{1}{2} h_{1}, \tag{2.32}
\end{equation*}
$$

where

$$
\begin{equation*}
h_{1}=m_{A} v_{0} \lambda_{1} \tag{2.32a}
\end{equation*}
$$

is the wave action of the fundamental tone. From these equalities, it follows that

$$
\begin{equation*}
h_{\lambda}=\frac{h_{1}}{n} \quad \text { or } \quad \hbar_{\lambda}=\frac{\hbar_{1}}{n} \text {, } \tag{2.33}
\end{equation*}
$$

where

$$
\begin{equation*}
\hbar_{\lambda}=\frac{h_{\lambda}}{2 \pi} \quad \text { and } \quad \hbar_{1}=\frac{h_{1}}{2 \pi} . \tag{2.33a}
\end{equation*}
$$

On the basis of formulae (2.33) and (2.33a), the kinetic energy of any atom of the string, at the level of the wave basis of the field, can be presented in the following forms:

$$
\begin{equation*}
\varepsilon_{k}=\frac{1}{2} h_{1} v_{1}=\hbar \omega_{1}=\frac{1}{2} \hbar_{1} \omega_{1}=\frac{m_{A} v_{0}^{2}}{2}, \tag{2.34}
\end{equation*}
$$

where

$$
\begin{equation*}
\hbar=\frac{m_{A} v_{0} l}{2 \pi}=\frac{1}{2} \hbar_{1}=\frac{1}{2} m_{A} v_{0} \lambda_{1} . \tag{2.34a}
\end{equation*}
$$

It is usual to call the measure (2.34) the energy quantum. The definite wave portions of any measures of potential-kinetic fields relate to quanta. In general, all quanta are effects of the wave nature of matter-space-time. Hence, strictly speaking, they should be called wave quanta, or quanta-periods. Such fundamental quanta of wave spaces, as quanta of the spatial extension of waves (represented by the wavelengths $\lambda$, the quanta of time length, the named periods of physical time waves $T$, etc.), relate to the wave quanta. Unfortunately, M. Born did not understand this and introduced the term "quantum mechanics", which began to press the wave theory of matter. It was a serious logical error.

The measure of energy (2.34) repeats the well-known Planck's energy for "quanta of energy - photons", which (as will be shown in the following chapters) do not exist in nature, being, at the most, mathematical fictions.

On the basis of all above stated, we can present the wave number squared, $k^{2}$, in the following form

$$
\begin{equation*}
k^{2}=\frac{\omega^{2}}{v_{0}^{2}}=\frac{2 m_{A} \varepsilon_{k}}{\hbar_{\lambda}^{2}} \tag{2.35}
\end{equation*}
$$

Then, the amplitude equation for the wave motion of the string (2.6) takes the form of Schrödinger's equation

$$
\begin{equation*}
\Delta \psi+\frac{2 m_{A} \varepsilon_{k}}{\hbar_{\lambda}^{2}} \psi=0 \tag{2.36}
\end{equation*}
$$

Because the quanta of mass at the atomic level, as W. Prout noted, are masses of $H$-atoms (neutrons, protons), hence, the equation. (2.36) can be presented as

$$
\begin{equation*}
\Delta \psi+\frac{2 m_{0} \varepsilon_{k}}{\hbar_{\lambda}^{2}} \psi=0 \tag{2.36a}
\end{equation*}
$$

where $m_{0}$ is the atomic mass unit, substituted in the above presented parameters for the mass of an atom of the string $m_{A}$.

Since the total potential-kinetic energy of basis in the wave field of the string, for every $H$-atom, is equal to $\varepsilon=\varepsilon_{k}+\varepsilon_{p}=\frac{m_{0} v_{0}^{2}}{2}+\frac{m_{0} v_{0}^{2}}{2} \varepsilon_{T}$, we can again rewrite the equation (2.36a) as

$$
\begin{equation*}
\Delta \psi+\frac{2 m_{0}\left(\varepsilon-\varepsilon_{p}\right)}{\hbar_{\lambda}^{2}} \psi=0 \tag{2.36b}
\end{equation*}
$$

In this form, the wave equation (2.36b) looks spectacularly. But actually, this equation is senseless. Here, $\frac{2 m_{0}\left(\varepsilon-\varepsilon_{p}\right)}{\hbar_{\lambda}^{2}}$ is the quantity, representing by itself a discrete series of values, which depend on the boundary conditions. Therefore, one should operate by the wave equation of superstructure in the form

$$
\Delta \psi+k^{2} \psi=0
$$

We can also present the wave equation of the harmonic $\Psi$-function (1.34) in the complex form:

$$
\frac{1}{v_{0}^{2}} \frac{\partial^{2} \Psi}{\partial t^{2}}=\frac{i \omega}{v_{0}^{2}} \frac{\partial \Psi}{\partial t} \quad \text { and } \quad \frac{\partial^{2} \Psi}{\partial z^{2}}=\frac{i \omega}{v_{0}^{2}} \frac{\partial \Psi}{\partial t}
$$

Then, multiplying the equality (1.34) by the wave action $\hbar_{\lambda}$, we have

$$
\frac{v_{0}^{2} \hbar_{\lambda}}{\omega} \Delta \Psi=\frac{v_{0} \hbar_{\lambda}}{k} \Delta \Psi=i \hbar_{\lambda} \frac{\partial \Psi}{\partial t}
$$

If we introduce the operator of the string

$$
\begin{equation*}
\hat{H}=\frac{v_{0} \hbar_{\lambda}}{k} \Delta=\frac{\hbar_{\lambda}^{2}}{m_{0}} \Delta \tag{2.37}
\end{equation*}
$$

then the equation of the string will have the form

$$
\begin{equation*}
\hat{H} \Psi=i \hbar_{\lambda} \frac{\partial \Psi}{\partial t} \tag{2.38}
\end{equation*}
$$

According to the fully developed opinion, in quantum mechanics, Schrödinger's wave equation in the form (2.38) is regarded as a particular equation, since it contains the first derivative of the wave function with respect to time and the imaginary unit. And because of this, its solution can be presented only by the complex wave $\Psi$-function. Of course, it is a fallacy. As has been shown above, the wave equation with the second derivative of the wave function with respect to time can be also presented with the first derivative of the same function with respect to time.

Previously, we devoted major attention, first of all, to the spatial $\psi(k z)$ function. Now, the time comes when we must turn to the detail analysis of the function $\hat{T}(\omega t)$ of the time field-space of the string.

The material field-space $\psi(k z)$ is characterized by the wave number of the space, $k$, connected with the wave quantum of the space $\lambda$ (the space wavelength) through the following ratio

$$
\begin{equation*}
\lambda=\frac{2 \pi}{k} \tag{2.39}
\end{equation*}
$$

In the time field-space $\hat{T}(\omega t)$, the wave number is the circular frequency $\omega$, which should be called the wave number of the time field-space. The time wave number $\omega$ is inseparable to the wave quantum of the time field - the period $T$. The period of the time wave is the time wavelength $T$, which is analogous to the space wavelength $\lambda$. Between the time wavelength $T$ and the time wave number $\omega$ takes place the ratio, analogous to (2.39):

$$
\begin{equation*}
T=\frac{2 \pi}{\omega} . \tag{2.40}
\end{equation*}
$$

$\hat{T}$-Function, in the explicit form, is presented as

$$
\hat{T}=e^{i \omega t}=\cos \omega t+i \sin \omega t
$$

In conformity with the potential and kinetic displacements

$$
x=a \sin k z \cos \omega t \text { and } \quad y=a \sin k z \sin \omega t
$$

we will call the real part of the time $\hat{T}$-function, $\operatorname{Re} \hat{T}$, the potential time displacement $t_{p}$, or simply, the potential time with the relative unit amplitude; the imaginary part, $\operatorname{Im} \hat{T}$, - the kinetic time displacement $t_{k}$, or simply, the kinetic time. In such a case, we have

$$
\begin{equation*}
\hat{T}=e^{i \omega t}=t_{p}+i t_{k}=\cos \omega t+i \sin \omega t \tag{2.41}
\end{equation*}
$$

where

$$
\begin{equation*}
t_{p}=\operatorname{Re} \hat{T}(\omega t)=\cos \omega t, \quad t_{k}=\operatorname{Im} \hat{T}(\omega t)=\sin \omega t \tag{2.41a}
\end{equation*}
$$

Thus, the physical time field is the potential-kinetic time wave; its graph is presented in Fig. 3.5.


Fig. 3.5. A graph of the time potential-kinetic wave: $t_{p}=\cos \omega t$ is the potential component and $t_{k}=\sin \omega t$ is the kinetic component of the wave of time.

The physical time wave field of the string is a particular case of the complicated ideal time wave field-space of the Universe with its infinite series of levels.

The following relation connects the spatial standing wave $\Psi$ (see Eq. (2.16) and Fig. 3.3) with the time wave:

$$
\begin{equation*}
\Psi=a \sin k z \cdot \hat{T}=a_{z} \hat{T} \tag{2.42}
\end{equation*}
$$

Therefore, they are structurally similar.
The rate of change of time wave is determined by the potential-kinetic time speed $\gamma$.

$$
\begin{equation*}
\gamma=\frac{d \hat{T}}{d t}=i \omega \hat{T}=\gamma_{k}+i \gamma_{p}=-\omega \sin \omega t+i \omega \cos \omega t \tag{2.43}
\end{equation*}
$$

where

$$
\begin{equation*}
\gamma_{k}=\frac{d t_{p}}{d t}=-\omega t_{k}=-\omega \sin \omega t \tag{2.43a}
\end{equation*}
$$

is the kinetic rate of change of time and

$$
\begin{equation*}
\gamma_{p}=\frac{d t_{k}}{d t}=\omega t_{p}=\omega \cos \omega t \tag{2.43b}
\end{equation*}
$$

is the potential rate of change of time.
Amplitude of the time speed is equal to the time wave number, referred to as the circular frequency (or, under the circular motion, the angular speed).

The following relation takes place between the spatial and time speeds of a standing wave:

$$
\begin{equation*}
v=\frac{d \Psi}{d t}=a \sin k z \frac{d \hat{T}}{d t}=a_{z} \cdot \gamma . \tag{2.44}
\end{equation*}
$$

At that,

$$
\begin{equation*}
v_{k}=a_{z} \cdot \gamma_{k}, i v_{p}=a_{z} \cdot i \gamma_{p} . \tag{2.45}
\end{equation*}
$$

According to the equations (2.29) and (2.30), the standing wave of amplitude energy of superstructure, at the level of $H$-masses, has the form

$$
\begin{equation*}
\varepsilon_{m}=\varepsilon_{k}-\varepsilon_{p}=\frac{m_{0} a_{z}^{2}}{2}\left(\gamma_{k}^{2}+\gamma_{p}^{2}\right)=\frac{m_{0} a_{z}^{2}}{2} \omega^{2} . \tag{2.46}
\end{equation*}
$$

The wave field of a string can be described also in the space of motion and time. As the measure of the space of motion, it is possible to take the wave fieldspace of speed $v(k z)$. If the time field-space is represented by the aforementioned function $\hat{T}(\omega t)$, then new $\Psi_{\nu}$-function takes the form

$$
\begin{equation*}
\Psi_{v}(z, t)=v(k z) \hat{T}(\omega t) \tag{2.47}
\end{equation*}
$$

This function satisfies the wave equation of a string for the wave field-space of speed and time:

$$
\begin{equation*}
\Delta \Psi_{v}=\frac{1}{v_{0}^{2}} \frac{\partial^{2} \Psi_{v}}{\partial t^{2}} \tag{2.48}
\end{equation*}
$$

Elementary solutions of the wave equation (2.48) define two traveling waves of speed, $\Psi_{v}^{+}$and $\Psi_{v}^{-}$, correspondingly, in positive and negative directions along the Z -axis.

Taking into account the boundary conditions, $\Psi_{v}^{+}$-function takes the form

$$
\begin{equation*}
\Psi_{v}^{+}=i \omega \Psi^{+}=i \omega A e^{i(\omega t-k z+\pi / 2)}=-\omega A e^{i(\omega t-k z)}=v_{k}^{+}+i v_{p}^{+} \tag{2.49}
\end{equation*}
$$

where

$$
\begin{align*}
& v_{k}^{+}=-\omega A \cos (\omega t-k z),  \tag{2.49a}\\
& v_{p}^{+}=-\omega A \sin (\omega t-k z) \tag{2.49b}
\end{align*}
$$

are the traveling waves of the kinetic and potential speeds, correspondingly.
The traveling wave of $\Psi_{v}^{-}$-speed, conjugated to $\Psi_{v}^{+}$, is

$$
\begin{equation*}
\Psi_{v}^{-}=i \omega \Psi^{-}=i \omega A e^{i(\omega t+k z-\pi / 2)}=\omega A e^{i(\omega t+k z)}=v_{k}^{-}+i v_{p}^{-}, \tag{2.50}
\end{equation*}
$$

where

$$
\begin{equation*}
v_{k}^{-}=\omega A \cos (\omega t+k z), \quad v_{p}^{-}=\omega A \sin (\omega t+k z) \tag{2.50a}
\end{equation*}
$$

On the basis of the equation (2.49), we obtain the expressions for the kinetic, potential, and amplitude energies of traveling $\Psi_{v}^{+}$-waves:

$$
\begin{gather*}
\varepsilon_{k}^{+}=\frac{m_{0}\left(v_{k}^{+}\right)^{2}}{2}=\frac{m_{0} \omega^{2} A^{2}}{2} \cos ^{2}(\omega t-k z)=\frac{m_{0} v_{m}^{2}}{2} \cos ^{2}(\omega t-k z)  \tag{2.51}\\
\varepsilon_{p}^{+}=\frac{m_{0}\left(i v_{p}^{+}\right)^{2}}{2}=-\frac{m_{0} \omega^{2} A^{2}}{2} \sin ^{2}(\omega t-k z)=-\frac{m_{0} v_{m}^{2}}{2} \sin ^{2}(\omega t-k z)  \tag{2.52}\\
\varepsilon_{m}^{+}=\varepsilon_{k}^{+}-\varepsilon_{p}^{+}=\frac{m_{0} v_{m}^{2}}{2} \tag{2.53}
\end{gather*}
$$

where $v_{m}=\omega A$ is the amplitude value of speed.
Potential and kinetic energies in traveling waves are shifted in phase by $90^{\circ}$ (Fig. 3.6a).


Fig. 3.6. A graph of the traveling (a) and standing (b) potential-kinetic waves of energy; $Q$ is the plane of cross-section of the standing wave, where potential-kinetic oscillations of the wave of time (c), shifted by $90^{\circ}$, occur.

Analogous wave energies defines $\Psi_{v}^{-}$-function:

$$
\begin{gather*}
\varepsilon_{k}^{-}=\frac{m_{0}\left(V_{k}^{-}\right)^{2}}{2}=\frac{m_{0} \omega^{2} A^{2}}{2} \cos ^{2}(\omega t+k z)=\frac{m_{0} v_{m}^{2}}{2} \cos ^{2}(\omega t+k z)  \tag{2.54}\\
\varepsilon_{p}^{-}=\frac{m_{0}\left(i V_{p}^{-}\right)^{2}}{2}=-\frac{m_{0} \omega^{2} A^{2}}{2} \sin ^{2}(\omega t+k z)=-\frac{m_{0} v_{m}^{2}}{2} \sin ^{2}(\omega t+k z)  \tag{2.55}\\
\varepsilon_{m}^{-}=\varepsilon_{k}^{-}-\varepsilon_{p}^{-}=\frac{m_{0} v_{m}^{2}}{2} \tag{2.56}
\end{gather*}
$$

Obviously, the sum of the traveling kinetic-potential waves, $\Psi_{v}^{+}$and $\Psi_{v}^{-}$, generates the standing wave of speed

$$
\begin{equation*}
\Psi_{v}=i \omega a \sin k z e^{i \omega t}=v_{k}+i v_{p} \tag{2.57}
\end{equation*}
$$

where

$$
\begin{equation*}
v_{k}=v_{k}^{+}+v_{k}^{-}=-\omega a \sin k z \sin \omega t=-\omega a_{z} \sin \omega t \tag{2.57a}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{p}=v_{p}^{+}+v_{p}^{-}=\omega a \sin k z \cos \omega t=\omega a_{z} \cos \omega t \tag{2.57b}
\end{equation*}
$$

Thus, the expressions for energies in the standing wave take the following form:

$$
\begin{gather*}
\varepsilon_{k}=\frac{m_{A} v_{k}^{2}}{2}=\frac{\hbar_{k} \omega}{2}=\frac{m_{A}(\omega a)^{2}}{2} \sin ^{2} k z \cdot \sin ^{2} \omega t,  \tag{2.58}\\
\varepsilon_{p}=\frac{m_{A}\left(i v_{p}\right)^{2}}{2}=-\frac{\hbar_{p} \omega}{2}=-\frac{m_{A}(\omega a)^{2}}{2} \sin ^{2} k z \cdot \cos ^{2} \omega t,  \tag{2.58a}\\
\varepsilon_{m}=\varepsilon_{k}-\varepsilon_{p}=\frac{\hbar \omega}{2}=\frac{m_{A} v_{m}^{2}}{2} \sin ^{2} k z, \tag{2.59}
\end{gather*}
$$

where $v_{m}=\omega a$ is the amplitude value of speed.
The structure of standing waves of potential-kinetic energy is shown in Fig. $3.6 \mathrm{~b}, \mathrm{c}$. These waves of energy were earlier presented by the formulae (2.20) and (2.20a).

If in a string, the two mutually perpendicular plane-polarized waves, traveling in opposite directions, are excited then the circular-polarized wave arises. The energy of every particle in this wave has now the constant value, independent of time:

$$
\begin{align*}
& \varepsilon_{k}=\frac{m_{A} v_{k}^{2}}{2}=\frac{m_{A} v_{m}^{2} \sin ^{2} k z}{2},  \tag{2.60}\\
& \varepsilon_{p}=\frac{m_{A}\left(i v_{p}\right)^{2}}{2}=-\frac{m_{A} v_{m}^{2} \sin ^{2} k z}{2} . \tag{2.60a}
\end{align*}
$$

From this we find the amplitude energy

$$
\begin{equation*}
\varepsilon_{m}=\varepsilon_{k}-\varepsilon_{p}=m_{A} v_{z}^{2}=\hbar_{z} \omega=m_{A} v_{m}^{2} \sin ^{2} k z . \tag{2.61}
\end{equation*}
$$

At that the total energy is equal to zero:

$$
\begin{equation*}
\varepsilon=\varepsilon_{k}+\varepsilon_{p}=0 \tag{2.62}
\end{equation*}
$$

Thus, every atom of a string, in such a wave, performs the circular motion with the total energy equal to zero. At that, the kinetic speed is directed tangentially to the circular trajectory, whereas the potential speed, perpendicular to the kinetic one, is directed to the Z-axis. In this wave, the kinetic and potential fields are mutually perpendicular (Fig. 3.7) and negate each other. Thus, it is clear why in nature the circular motion prevails.


Fig. 3.7. A circular potential-kinetic wave of the string.

Obviously, the description of the wave field can be realized also by the complex $\Psi$-function, which represents other physical parameters, such as the potential-kinetic momentum, potential-kinetic kinema, etc. It means that the wave $\Psi$-function of string can describe any potential-kinetic fields of the string: fields of displacements, speeds-time, momenta-time, etc. All these wave fields represent by themselves the different measures of a many-sided potential-kinetic field of a string.

In virtue of the above stated, in dialectics, the question about the interpretation of the wave $\Psi$-function makes no logical sense, because everything depends on which facets of the potential-kinetic field we mean to regard. If we would write the equation of a wave process, or a wave system, only formally and, further, be engaged in scientific guessing ("interpretation"), then we could come to the creation of myths. And myths, as is well known, take deep roots in the mass scientific consciousness, making it blind.

In this connection, let us once more turn to the generalized equation of the string (2.36), introducing in it some potential energy of interaction of atoms $\varphi(z)$ (that, unconditionally, is inadmissible; we have written about it already). Then, designating the total energy of oscillations of an atom of the string in a cross-section $S$ by the letter $W$, we can write:

$$
\begin{equation*}
\varepsilon_{k}=W-\varphi(z) \tag{2.63}
\end{equation*}
$$

On the basis of such a "generalization", the wave number

$$
\begin{equation*}
k=\frac{\omega}{v_{0}}=\sqrt{\frac{2 m \varepsilon_{k}}{\hbar_{0}^{2}}}=\sqrt{\frac{2 m}{\hbar_{0}^{2}}(W-\varphi(z))} \tag{2.64}
\end{equation*}
$$

and, corresponding to it, frequency of the wave field of the string

$$
\begin{equation*}
\omega=v_{0} \sqrt{\frac{2 m}{\hbar_{0}^{2}}(W-\varphi(z))} \tag{2.65}
\end{equation*}
$$

will be the functions of the coordinate $z$ of points of the string, $k=k(z)$ and $\omega=\omega(z)$. Actually, the wave number $k$ is the constant quantity, defining some frequency of the wave field $\omega$, which bonds the wave system in a single whole.

According to such a "generalization", all points of the string must oscillate with different frequencies. This is an absurdity, possible only due to a free game of notions, unrestricted within the reasonable framework of logic.

This cardinal error has transformed quantum mechanics into a great caricature about the world of real wave processes. And the extensive publicity created an illusion as if mankind deals with a great theory. In fact, a phenomenological theory, with the definite fitting of it to the experiment, was built as a result. This theory (quantum mechanics) as much significantly distorted the real picture of the microworld that it became the world of theoretical monsters and quantum chaos, but not the world of real images.

The absurdity of the above-described formal "deducing" of the relation (2.64) (for the field of the string) is clear and no sane physicist will agree with the "generalized" wave equation of the string in the form as follows:

$$
\begin{equation*}
\Delta \psi+\frac{2 m}{\hbar_{0}^{2}}(W-\varphi(z)) \psi=0 \tag{2.66}
\end{equation*}
$$

The falsity of such a formal generalization-derivation is obvious. But in the 1930's, the similar work for the hydrogen atom was (and still) declared as a greatest penetration in the mystery of the atomic world. Many physicists have objected to such a theory, but they had (and yet have) no apparatus to be able to "see" atoms; therefore, it was very difficult to object to the aggressive physicalmathematical decadence. And the decadence overcame reason. Since then, all right notices against such a formalistic arbitrariness were/are declared as the "classical backwardness".

## 3. Parameters of the wave field of a string and Balmer's formula

The wave motion of a string with the frequency of the fundamental tone $v_{1}$ and wavelength $\lambda_{1}$ generates an acoustic wave in the surrounding air of the same frequency:

$$
\begin{equation*}
v_{1}=\frac{1}{T_{1}}=\frac{v_{0}}{\lambda_{1}}=\frac{c}{\lambda_{c}}, \tag{3.1}
\end{equation*}
$$

where $\lambda_{c}$ is the acoustic wavelength, $c$ is the speed of sound. From the equations (2.13) and (3.1), the following relation for the fundamental tone takes place:

$$
\begin{equation*}
\frac{1}{\lambda_{c}}=\frac{v_{0} / c}{2 l} . \tag{3.2}
\end{equation*}
$$

Let us imagine that the ends of the string are joined together, i.e., we deal with a string circle with one node. If the radius of the string circle is equal to $r_{1}$ then the equation (3.2) will take the form

$$
\begin{equation*}
\frac{1}{\lambda_{c}}=\frac{v_{0} / c}{4 \pi r_{1}}=\frac{v_{1}}{c} \tag{3.3}
\end{equation*}
$$

where

$$
\begin{equation*}
v_{1}=\frac{v_{0}}{4 \pi r_{1}} \tag{3.4}
\end{equation*}
$$

is the frequency of the fundamental tone of the string.
If we have a series of string circles of the length $l_{n}$, whose radii $r_{n}$ are multiple to the radius of the first circle $r_{1}, r_{n}=n r_{1}$, and the wave speeds are inversely
proportional to numbers of this series, $v_{n}=v_{1} / n$, then such strings will have the following proper frequencies:

$$
\begin{equation*}
v_{n}=\frac{v_{0}}{4 \pi r_{1} n^{2}} . \tag{3.5}
\end{equation*}
$$

These frequencies will define the spectrum of acoustic waves of the fundamental tone

$$
\begin{equation*}
\frac{1}{\lambda_{n}}=\frac{v_{0} / c}{4 \pi r_{1}} \frac{1}{n^{2}} . \tag{3.6}
\end{equation*}
$$

The difference of the two inverse acoustic wavelengths (two arbitrary string levels of (3.6)) is presented by the following equality:

$$
\begin{equation*}
\Delta\left(\frac{1}{\lambda}\right)=\frac{v_{0} / c}{4 \pi r_{1}}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right)=R\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right), \tag{3.7}
\end{equation*}
$$

where

$$
\begin{equation*}
R=\frac{v_{0} / c}{4 \pi r_{1}} \tag{3.8}
\end{equation*}
$$

is the wave constant of the string of the fundamental tone.
Let us remember now Balmer's wave formula for H -atom spectrum

$$
\begin{equation*}
\frac{1}{\lambda}=R\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right)=\frac{v_{0} / c}{4 \pi r_{0}}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right), \tag{3.9}
\end{equation*}
$$

where $V_{0}$ and $r_{0}$ are the Bohr speed and radius, correspondingly. Obviously, for the limiting wave of radiation, of the fundamental tone of $H$-atom ( $n=\infty$ and $m=1$ ), the following relation, analogous to the relation for the string (3.8), takes place:

$$
\begin{equation*}
\frac{1}{\lambda}=R=\frac{v_{0} / c}{4 \pi r_{0}}=\frac{v_{1}}{c}, \tag{3.10}
\end{equation*}
$$

where

$$
\begin{equation*}
v_{1}=\frac{1}{T_{1}}=\frac{v_{0}}{4 \pi r_{0}} \tag{3.11}
\end{equation*}
$$

is the frequency of the fundamental tone of the first electron orbit.
Conformity of the parameters, for the electron orbit and string circle, is obvious. And it is natural, because an electron on its orbit is the node of the electron circular cylindrical wave. A physical mystery of Rydberg constant is revealed here. The relation (3.10) has the fundamental wave character. It points out the existence of a series of Rydberg constants for many wave systems. In a particular case of a string, the Rydberg acoustic constant is

$$
\begin{equation*}
R=\frac{v_{0} / c}{2 l} . \tag{3.12}
\end{equation*}
$$

Thus, the formula of the Rydberg constant uniquely rejects the quantum mechanical myth that electrons in atoms have no orbits (trajectories) and their probabilistic chaotic motion forms so-called "orbitals" ("charged clouds" of probability of electron's localization). All this is formalistic fantasy, yet we will analyze it in detail in the following chapters.

Earth's orbit is analogous to the electron orbit of the fundamental tone. Therefore, the formula (3.10) is also valid for the wave gravitational field of the solar system. Since the period of Earth's rotation is equal to one-half period of the fundamental tone of Earth's orbit (with its only gravitational node-Earth), therefore, it should be assumed the existence in our Galaxy of waves in two light years and half-waves in one light year. And actually, a half-wave of Earth's orbit shows its worth in the distances between stars.

The sand grains in Chladni's famous experiments * with oscillating plates are localized not in points of maxima of $\Psi$-function, but only in points of its zero values, i.e., in the nodal points of the field. Just so, we should assume that stars, in the galactic wave field, are situated in nodes of cosmic space. The astronomical data confirms this supposition. Bessel, who has measured the parallax of stars, carried out (in 1838) the first reliable determination of the distance to the star 61, Swan. It proven to be 11 light years (the calculated value, according to the parallax in $0.296^{\prime \prime}$, is 11.0247 ). Bernard's star has the parallax in $0.543^{\prime \prime}$ that corresponds to the distance in 6 light years (calculations give 6.0016). The star Procaine: the parallax is $0.297^{\prime \prime}$, the distance -11 light years (10.9921), etc. Thus, within accuracy of measurements of parallaxes, the distances to the abovementioned stars are multiple, precisely enough, to a half-wave of Earth's orbit.

A string, fixed from both ends, can be called (in a definite sense) the closed string. If one end of the string is fixed and another one is free (open), it will be then a half-open string. In such a string, an elementary standing wave also arises,

$$
\begin{equation*}
\Psi=\Psi^{-}+\Psi^{+}=a \cos \left(k z-\varphi_{0}\right) e^{i a t} . \tag{3.13}
\end{equation*}
$$

The boundary conditions for the fixed and free ends are

$$
\begin{align*}
& \operatorname{Re} \Psi(k z, t)=\left.a \cos \left(k z-\varphi_{0}\right)\right|_{z=0}=0,  \tag{3.14}\\
& \operatorname{Re} \Psi(k z, t)=a \cos \left(k z-\varphi_{0}\right)_{z=l}=d, \tag{3.15}
\end{align*}
$$

From the condition (3.14), we find the initial phase, $\varphi_{0}=\pi / 2$. The second boundary condition (3.15) gives the possible values of the wave number and frequency of the field of string:

$$
\begin{equation*}
k=\frac{1}{l} \arcsin \left(\frac{d}{a}\right) . \tag{3.16}
\end{equation*}
$$

[^1]Let us assume that displacements of the free end of the string take the following series of values

$$
\begin{equation*}
\arcsin \left(\frac{d}{a}\right)=\pi\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right), \tag{3.17}
\end{equation*}
$$

where $m$ and $n$ are integers. This condition always can be realized. The discrete series of values of the wave number and circular frequency corresponds to the above condition:

$$
\begin{equation*}
k=\frac{\pi}{l}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right), \quad \omega=\frac{\pi v_{0}}{l}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right) . \tag{3.18}
\end{equation*}
$$

As follows from (3.18), standing waves of the undertones must arise in the half-open string:

$$
\begin{equation*}
\Psi=a \sin \left(\frac{\pi}{l}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right) z\right) \exp \left(i \frac{\pi v_{0}}{l}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right) t\right) . \tag{3.19}
\end{equation*}
$$

Oscillations of the open string excite acoustic waves in the air of the same frequency

$$
\begin{equation*}
v=\frac{v_{0}}{\lambda}=\frac{c}{\lambda_{c}}=\frac{v_{0}}{2 l}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right) ; \tag{3.20}
\end{equation*}
$$

they form the acoustic spectrum of waves, analogous to the $H$-atom spectrum:

$$
\begin{equation*}
\frac{1}{\lambda_{c}}=\frac{v_{0}}{2 c l}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right) . \tag{3.21}
\end{equation*}
$$

## 4. The mathematical basis of the hydrogen atom using the wave equation of matter-space-time

Quantum mechanics, with its Schrödinger's $\Psi$-function, has actually no relation to the concrete presentation of the microworld. The philosophy of abstractionism lies in its base. Abstractionism exhibited itself first in art. As usual to regard, the first work of abstractionism was Candinsky's picture of 1910 (in wa-ter-color), painted in Munich. Ch. Bru, a French critic, has wrote, "For the first time in history of painting, it is impossible to find or recognize anything in this picture" *. Bru noted also that Candinsky's abstractive idea arose when he, by chance, had leaned one of his pictures, looking at it under the angle: the image on this picture has became incomprehensible, as much enigmatically paints have sparkled at once. According to another version (by A.A. Sidorov): one day, Candinsky, drying the brushes, paid attention to the combination of accidental stains on a rag and thought, for a moment, why not regard this rag as the work of abstract art, without any objective content.

[^2]According to theorists of the abstract view on the world, abstract painting is painting "... when we can know nothing from it about the objective reality, being the normal sphere of our life... From this it follows that abstract painting is a reproduction brought to such an extreme, when it is impossible to find there an initial natural object; it is the reproduction, which, at first sight on it, nothing reminds us of the original. Only this work will have the right to be called the abstraction" *.

The physical-mathematical abstractionists have acted in the same spirit. Their philosophy sweeps aside, without a trace, any possibility of the concrete presentation of the microworld, in full agreement with Mach's and Ostwald's (F.W. Ostwald, 1853-1932, German chemist) ideas. Ostwald has written, in 1902, that the time, when all these "atoms and molecules will disappear in a dust of archives", is not so far. Mach has called the electron theory as a "respectable witches' sabbath".

Therefore, it is no wonder that, in quantum mechanics, the extraordinary primitive abstract model, based on the formal mathematical group of waves, represents the correlation of waves and particles. The superposition of these waves forms an envelope, spreading in space with the speed, different from the speed of constituent waves. This envelope, called the wave packet, is regarded as a microparticle; and waves, related to the packet, as waves of probability. Thus, the foundation of nature is represented as probabilistic "vague" chaos.

We will present now, in general outline, the abstract-concrete dialectical description of the microworld, which reflects their authors' works **.

The wave equation of matter-space-time

$$
\begin{equation*}
\Delta \Psi=\frac{1}{v^{2}} \frac{\partial^{2} \Psi}{\partial t^{2}} \tag{4.1}
\end{equation*}
$$

lies at the base of the dialectical description of the world. $\Psi$-Function represents the mathematical image of the potential-kinetic field of space-time of the atomic level (the level of superstructure over the nearest subatomic level - the level of basis). The wave equation (4.1) falls, like Schrödinger's equation, into the equation of spatial $\psi$-function,

$$
\begin{equation*}
\Delta \psi+k^{2} \psi=0 \tag{4.2}
\end{equation*}
$$

and the equation of time $\hat{T}$-function,

$$
\begin{equation*}
\frac{d^{2} \hat{T}}{d t^{2}}=-\omega^{2} \hat{T} \tag{4.2a}
\end{equation*}
$$

$\hat{T}$-Function has the form

$$
\begin{equation*}
\hat{T}=e^{i \omega t} \tag{4.2b}
\end{equation*}
$$

[^3]
### 4.1. The spherical wave field of $\mathbf{H}$-atom

If a wave object and its wave field-space have the spherical structure, that is characteristic for all microparticles, then the elementary structure of the field of space-time of microobjects is expressed by the functions

$$
\begin{equation*}
\Psi_{l, m}=\psi_{l, m}(\rho, \theta, \varphi) \hat{T}(\omega t)=R_{l}(\rho) \Theta_{l, m}(\theta) \Phi_{m}(\varphi) \hat{T}(\omega t), \tag{4.3}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi_{l, m}(\rho, \theta, \varphi)=R_{l}(\rho) \Theta_{l, m}(\theta) \Phi_{m}(\varphi) \tag{4.3a}
\end{equation*}
$$

is the spatial $\psi_{l, m}$-function, $\rho=k r$ is the relative radius, $k$ is the wave number of atomic space, and $\hat{T}(\omega t)$ is the time function.

The structure of $\Psi$-functions (4.3) allows decomposing the spatial equation (4.2) into three equations:

$$
\begin{align*}
& \frac{d^{2} R}{d \rho^{2}}+\frac{2}{\rho} \frac{d R}{d \rho}+\left(1-\frac{l(l+1)}{\rho^{2}}\right) R=0  \tag{4.4}\\
& \frac{d^{2} \Theta_{l, m}}{d \theta^{2}}+\operatorname{ctg} \theta \frac{d \Theta_{l, m}}{d \theta}+\left(l(l+1)-\frac{m^{2}}{\sin ^{2} \theta}\right) \Theta_{l, m}=0  \tag{4.4a}\\
& \frac{d^{2} \Phi_{m}}{d \varphi^{2}}=-m^{2} \Phi_{m} \tag{4.4b}
\end{align*}
$$

where $\rho=k r$ is the radial argument and $k=\frac{2 \pi}{\lambda}$ is the wave radial number of the atomic space. The azimuth function has the form

$$
\begin{equation*}
\Phi_{m}(\varphi)=e^{ \pm i\left(m \varphi+\varphi_{0}\right)} \tag{4.4c}
\end{equation*}
$$

Here, $\varphi_{0}$ is the initial phase, which is defined on the basis of the azimuth structure of a microobject.

The simplest polar-azimuth functions, within the constant factor, are presented in Table 3.1. The initial phase $\varphi_{0}$, for simplicity, is omitted.

Solutions of the wave equation (4.2) differ from the solutions of Schrödinger's equation because of their radial components, which are different, in principle. Elementary radial solutions of the atomic space do not diverge. They are presented by Bessel spherical functions, well known in the theory of wave fields. Only two wave numbers, $l$ and $m$, characterize elementary functions of wave atomic space.

In the simplest case of the spherical field, elementary radial solutions are described by the radial functions of half-integer order:

$$
\begin{equation*}
\hat{R}_{l}(\rho)=\frac{A \hat{e}_{l}(\rho)}{\rho} \tag{4.5}
\end{equation*}
$$

where

$$
\begin{equation*}
\hat{e}_{l}(\rho)=\sqrt{\frac{\pi \rho}{2}} H_{l+1 / 2}^{ \pm}(\rho)=\sqrt{\frac{\pi \rho}{2}}\left(J_{l+1 / 2}(\rho) \pm i N_{l+1 / 2}(\rho)\right) \tag{4.5a}
\end{equation*}
$$

is the spherical "exponent".
Table 3.1. Polar-azimuth functions.

| $l$ | $m$ | $\Theta_{l, m}(\theta) \Phi_{m}(\varphi)$ |
| ---: | ---: | :--- |
| 0 | 0 | 1 |
| 1 | 0 | $\cos \theta$ |
|  | $\pm 1$ | $\sin \theta \exp ( \pm i \varphi)$ |
| 2 | 0 | $\cos ^{2} \theta-1 / 3$ |
|  | $\pm 1$ | $\sin \theta \cos \theta \exp ( \pm i \varphi)$ |
|  | $\pm 2$ | $\sin ^{2} \theta \exp ( \pm 2 i \varphi)$ |
| 3 | 0 | $\cos \theta\left(\cos ^{2} \theta-3 / 5\right)$ |
|  | $\pm 1$ | $\sin \theta\left(\cos ^{2} \theta-1 / 5\right) \exp ( \pm i \varphi)$ |
|  | $\pm 2$ | $\sin ^{2} \theta \cos ^{2} \theta \exp ( \pm 2 i \varphi)$ |
|  | $\pm 3$ | $\sin ^{3} \theta \exp ( \pm 3 i \varphi)$ |
| 4 | 0 | $\cos ^{4} \theta-6 / 7 \cos ^{2} \theta+3 / 35$ |
|  | $\pm 1$ | $\sin ^{2} \theta \cos \theta\left(\cos ^{2} \theta-3 / 7\right) \exp ( \pm i \varphi)$ |
|  | $\pm 2$ | $\sin ^{2} \theta(\cos 2 \theta-1 / 7) \exp ( \pm 2 i \varphi)$ |
|  | $\pm 3$ | $\sin ^{3} \theta \cos \theta \exp ( \pm 3 i \varphi)$ |
|  | $\pm 4$ | $\sin ^{4} \theta \exp ( \pm 4 i \varphi)$ |
| 5 | 0 | $\cos ^{2} \theta\left(\cos { }^{4} \theta-10 / 9 \cos ^{2} \theta+5 / 21\right)$ |
|  | $\pm 1$ | $\sin ^{2} \theta\left(\cos { }^{4} \theta-2 / 3 \cos ^{2} \theta+1 / 21\right) \exp ( \pm i \varphi)$ |
|  | $\pm 2$ | $\sin ^{2} \theta \cos \theta\left(\cos ^{2} \theta-1 / 3\right) \exp ( \pm 2 i \varphi)$ |
|  | $\pm 3$ | $\sin ^{3} \theta\left(\cos { }^{2} \theta-1 / 9\right) \exp ( \pm 3 i \varphi)$ |
|  | $\pm 4$ | $\sin ^{4} \theta \cos \theta \exp ( \pm 4 i \varphi)$ |
|  | $\pm 5$ | $\sin ^{5} \theta \exp ( \pm 5 i \varphi)$ |

Considering the solutions of the spherical field (4.5), one should keep in mind that the argument $\rho$ can take values within the interval from $\rho_{1}=k r_{0}$ to $\rho=\infty$. The radial parameter $r_{0}$ is the radius of the sphere-shell, separating the proper space of $H$-atom (with its "atmosphere") from the surrounding field-space of matter. Thus, $\boldsymbol{H}$-atom is regarded as the atom of the field of mat-ter-space-time. Its shell of the radius $r_{0}$ is the boundary shell of the wave atomic space, from below, while the upper boundary shell is boundless.

At $l=0$, we obtain a simplest solution

$$
\begin{equation*}
\psi_{0,0}(\rho)=\frac{A \hat{e}_{0}(\rho)}{\rho}=\frac{A \sqrt{\frac{\pi \rho}{2}} H_{1 / 2}^{+}(\rho)}{\rho}=\frac{A(\sin \rho+i \cos \rho)}{\rho} \tag{4.6}
\end{equation*}
$$

where

$$
\begin{equation*}
\operatorname{Re} \psi_{0,0}(\rho)=\frac{A \sin \rho}{\rho} \tag{4.6a}
\end{equation*}
$$

is the potential displacement and

$$
\begin{equation*}
\operatorname{Im} \psi_{0,0}(\rho)=\frac{A \cos \rho}{\rho} \tag{4.6b}
\end{equation*}
$$

is the kinetic displacement. The condition

$$
\begin{equation*}
\operatorname{Re} \psi_{0,0}(k r)=\frac{A \sin k r}{k r}=0 \tag{4.7}
\end{equation*}
$$

defines the radii of potential spheres (shells), situated from each other at the distance of a radial half-wave,

$$
\begin{equation*}
k r=n \pi \quad \text { or } \quad r=n \frac{\lambda_{r}}{2} \tag{4.8}
\end{equation*}
$$

and the radii of potential spheres, multiple to one wave,

$$
\begin{equation*}
k r=2 \pi n \quad \text { or } \quad r=n \lambda_{r} . \tag{4.8a}
\end{equation*}
$$

On the boundary shell, the condition (4.8) takes the form

$$
\begin{equation*}
r_{0}=\frac{\lambda_{r}}{2} \quad \text { or } \quad \lambda_{r}=2 r_{0} \tag{4.8b}
\end{equation*}
$$

Hence, the radii of stationary shells turn out to be multiple to the radius of the boundary shell,

$$
\begin{equation*}
r=n \frac{\lambda_{r}}{2}=r_{0} n \tag{4.9}
\end{equation*}
$$

A radial wave of the boundary shell defines the azimuth wave of the shell,

$$
\begin{equation*}
\lambda_{0}=2 \pi \lambda_{r}=4 \pi r_{0} . \tag{4.10}
\end{equation*}
$$

Thus, the elementary wave $\lambda_{0}$ is the wave of the fundamental tone. Similar to the case of the wave field of a string, only a half-wave of the fundamental tone is placed on the electron orbit, and the electron is in the node of the wave.

The condition (4.9) defines the spherical wave field with the uniformly disposed potential spheres.

It should be noted, in this connection, that other solutions of the equation (4.5), unrelated to $H$-atom, with wave numbers $l \neq 0$ (we will discuss it in other chapters), define the discrete nodal structure of the others atoms. From these solutions it follows that the spherical space of the atoms is nonuniform. Roots of the Bessel functions determine radii of their stationary nodal spheres:

$$
\begin{equation*}
\operatorname{Re} \Psi=\sqrt{\frac{\pi \rho}{2}} J_{l+1 / 2}(\rho)=0 . \tag{4.11}
\end{equation*}
$$

If we will denote the roots of the order $v=l+1 / 2$ by $z_{v, s}$, where $s$ is the number of the root, then we have

$$
\begin{equation*}
k r=z_{v, s} \quad \text { or } \quad r=\frac{z_{v, s}}{\pi} \frac{\lambda_{r}}{2} \tag{4.12}
\end{equation*}
$$

Under $k r \gg 1$, the condition (4.11) is substituted for the approximate equality

$$
\begin{equation*}
\operatorname{Re} \Psi=\sqrt{\frac{\pi \rho}{2}} \cdot \sqrt{\frac{2}{\pi \rho}} \cos \left(\rho-\frac{v \pi}{2}-\frac{\pi}{4}\right)=0 \tag{4.13}
\end{equation*}
$$

and

$$
\begin{equation*}
k r-\frac{v \pi}{2}-\frac{\pi}{4}=\frac{\pi}{2}(2 s-1) \quad \text { or } \quad r=\left(\frac{l}{2}+s\right) \frac{\lambda_{r}}{2} \tag{4.14}
\end{equation*}
$$

For $l \ll s$, we have

$$
\begin{equation*}
r=s \frac{\lambda_{r}}{2} \tag{4.15}
\end{equation*}
$$

Thus, for $l \neq 0$, the spherical wave field-space approaches to the uniform one only at large distances.

Let us return again to the solutions for $H$-atom $(l=0)$. According to Eq. (4.6), $\Psi$-function of the spherical field of H -atom has the form

$$
\begin{equation*}
\Psi^{+}=\psi_{0,0}(\rho) e^{i \omega t}=\frac{i A e^{i(\omega t-\rho)}}{\rho}=\frac{i A e^{i(\omega t-k r)}}{k r} \tag{4.16}
\end{equation*}
$$

where the superscript " + " of the wave function shows that the traveling spherical wave moves in the positive direction, in which it spreads in all directions from $H$-atom. The corresponding convergent wave has the structure

$$
\begin{equation*}
\Psi^{-}=\psi_{0,0}(\rho) e^{i \omega t}=\frac{i A e^{i(\omega t+\rho)}}{\rho}=\frac{i A e^{i(\omega t+k r)}}{k r} \tag{4.16a}
\end{equation*}
$$

The sum of waves, (4.16) and (4.16a), defines the standing wave, in space, with respect to time:

$$
\begin{equation*}
\Psi=\Psi^{+}+\Psi^{-}=\frac{2 i A \cos \rho \cdot e^{i \omega t}}{\rho}=\frac{2 i A \cos k r \cdot e^{i \omega t}}{k r}=\frac{i a \cos k r \cdot e^{i \omega t}}{k r} \tag{4.17}
\end{equation*}
$$

where $a=2 A$ is the amplitude of the standing wave.
At the potential shells $(k r=n \pi)$, the kinetic and potential amplitudes are equal, correspondingly, to

$$
\begin{equation*}
a_{m}=\frac{i a}{k r}, \quad a_{m}=-\frac{a}{k r} \tag{4.18}
\end{equation*}
$$

Within the signs of directions of displacements, $\pm$, and the imaginary unit $i$ (the indicator of belonging of displacements to the kinetic field), the values (modules) of potential and kinetic amplitudes are equal to the same numerical measure

$$
\begin{equation*}
a_{m}=\frac{a}{k r} \tag{4.18a}
\end{equation*}
$$

Sometimes, when only these will interest us, we will operate only with modules of amplitudes. For the intratomic field, $k=\omega / v_{0}$ and the amplitude of displacement in the spherical field of $\Psi$-function has the form

$$
\begin{equation*}
a_{m}=\frac{v_{0} a}{\omega r} . \tag{4.19}
\end{equation*}
$$

The amplitude of displacement defines the amplitude kinetic and potential speeds

$$
\begin{equation*}
v_{m}=\frac{i \omega i a}{\rho}=-\frac{v_{0} a}{r}, \quad v_{m}=\frac{i \omega a}{\rho}=\frac{i v_{0} a}{r} \tag{4.20}
\end{equation*}
$$

The sign "-" indicates the direction of motion at the instant, when the phase of the wave is equal to zero, $\omega t-k r=0$; therefore, it does not matter and can be omitted for the sake of simplicity.

In the spherical wave field, the amplitude elementary momentum of a particle of mass $m$ is

$$
\begin{equation*}
p_{m}=m v_{m}=\frac{m v_{0} a}{r} . \tag{4.21}
\end{equation*}
$$

From this we find the elementary wave action

$$
\begin{equation*}
\hbar=p_{m} r=m v_{m} r=m v_{0} a \tag{4.22}
\end{equation*}
$$

If $a=r_{0}$ and $v_{0}$ is the speed on the first Bohr orbit, then this action is Planck's action. The radial action (4.22) defines the azimuth action

$$
\begin{equation*}
h=2 \pi \hbar=p_{m} 2 \pi r=m v_{m} 2 \pi r=2 \pi m v_{0} a . \tag{4.22a}
\end{equation*}
$$

As we see, the elementary actions in the spherical field are the constant parameters of the field.

According to the equations (4.10) and (4.22a), the wave action of the wave of the fundamental tone $h_{\lambda}$ will be presented as

$$
\begin{equation*}
h_{\lambda}=4 \pi \hbar=4 \pi m v_{0} a . \tag{4.23}
\end{equation*}
$$

Obviously, azimuth and wave actions of the wave of fundamental tone are related as

$$
\begin{equation*}
h=\frac{1}{2} h_{\lambda}=2 \pi m v_{0} r_{0} . \tag{4.23a}
\end{equation*}
$$

The elementary momentum (4.21) allows determining the potential energy of an arbitrary particle of this field with the mass $m$ :

$$
\begin{equation*}
E=\frac{p_{m}^{2}}{2 m}=\frac{m v_{m}^{2}}{2}=-\frac{m a^{2} v_{0}^{2}}{2 r^{2}} \tag{4.24}
\end{equation*}
$$

According to the condition (4.8), energies of particles on the potential spheres are

$$
\begin{equation*}
E=\frac{p_{m}^{2}}{2 m}=-\frac{2 m a^{2} v_{0}^{2}}{\lambda_{r}^{2}} \frac{1}{n^{2}} \tag{4.24a}
\end{equation*}
$$

where $a=2 A$ is the amplitude in the standing wave. The factor 2 , in the denominator, shows that under other equal conditions the energy is half as much the energy of circular motion, the motion with two degrees of freedom resulted in the superposition of two mutually orthogonal potential-kinetic waves.

In the traveling wave, a distance between two sequential potential shells, in a simplest case, is equal to the amplitude of oscillations $a$. On the other hand, this distance is equal to a half-wave, hence,

$$
\begin{equation*}
a=2 A=\lambda_{r} / 2 . \tag{4.25}
\end{equation*}
$$

In particular, on the boundary shell, we have

$$
\begin{equation*}
r_{0}=a=2 A=\lambda_{r} / 2 . \tag{4.25a}
\end{equation*}
$$

Taking into account the equation (4.25), the expression (4.24a) can be rewritten as

$$
\begin{equation*}
E=\frac{p_{m}^{2}}{2 m}=\frac{m v_{m}^{2}}{2}=-\frac{m v_{0}^{2}}{2} \frac{1}{n^{2}} . \tag{4.26}
\end{equation*}
$$

### 4.2. The cylindrical wave field of $\mathbf{H}$-atom

An electron, moving along the wave orbit, represents by itself the cylindrical wave field. Therefore, its motion should be described by the wave equation (4.1) and for the cylindrical wave field-space.

We will suppose that the plane of the orbiting electron is perpendicular to $Z$-axis, then $\Psi$-function can be presented as

$$
\begin{equation*}
\Psi_{l}=\psi_{l}(\rho, \varphi, z) \hat{T}(\omega t)=R_{l}(\rho) \Phi_{l}(\varphi) \hat{Z}\left(k_{z} z\right) \hat{T}(\omega t) \tag{4.27}
\end{equation*}
$$

In the cylindrical field, the wave equation (4.2) falls into the following equations:

$$
\begin{gather*}
\frac{d^{2} R}{d \rho^{2}}+\frac{1}{\rho} \frac{d R}{d \rho}+\left(1-\frac{l^{2}}{\rho^{2}}\right) R=0,  \tag{4.28}\\
\frac{d^{2} \Phi}{d \varphi^{2}}=-m^{2} \Phi,
\end{gathered} \begin{gathered}
\frac{d^{2} \hat{Z}}{d z^{2}}=-k_{z}^{2} \hat{Z}, \tag{4.29}
\end{gather*} \frac{d^{2} \hat{T}}{d t^{2}}=-\omega^{2} \hat{T},
$$

where $k_{z}$ is the wave number of the wave, continuing along the Z-axis.
Thus, the elementary space component can be presented by the following space-wave

$$
\begin{equation*}
\psi_{l}(\rho, \varphi, z)=R_{l}(\rho) e^{-i l \varphi} e^{-i k_{z} z} \tag{4.30}
\end{equation*}
$$

Simplest solutions of the radial equation (4.28) are

$$
\begin{equation*}
R_{l}(\rho)=A \sqrt{\frac{\pi}{2}}\left(J_{l}(\rho) \pm i N_{l}(\rho)\right) . \tag{4.31}
\end{equation*}
$$

As a result, the cylindrical wave function takes the form

$$
\begin{equation*}
\Psi_{l}=A \sqrt{\frac{\pi}{2}}\left(J_{l}(\rho) \pm i N_{l}(\rho)\right) e^{-i\left(l \varphi+\varphi_{0}\right)} e^{-i k_{z} z} e^{i \omega t} \tag{4.32}
\end{equation*}
$$

where $\varphi_{0}$ is the initial phase of the azimuth wave.
In the cylindrical field, the order of the radial function $l$ defines the number of waves, which are placed on the orbit. In a simplest case, one electron, as the node of the wave, shows that only half-wave is placed on its orbit. So that such an orbit will be described by the function of the order $l=1 / 2$. As a solution, we choose the following function

$$
\begin{equation*}
\Psi_{1 / 2}=A \sqrt{\frac{\pi}{2}}\left(J_{1 / 2}(\rho)+i N_{1 / 2}(\rho)\right) e^{-i\left(1 / 2 \varphi+\varphi_{0}\right)} e^{-i k_{z} z} e^{i \omega t} \tag{4.33}
\end{equation*}
$$

or

$$
\begin{equation*}
\Psi_{1 / 2}^{+}=A i \frac{e^{i(\omega t-k r)}}{\sqrt{k r}} e^{-i\left(1 / 2 \varphi+\varphi_{0}\right)} e^{-i k_{z} z}, \tag{4.34}
\end{equation*}
$$

where $\varphi_{0}$ is the initial phase of the azimuth component of the radial divergent wave, defined on the basis of the boundary conditions. Naturally, the "radial divergent wave" is not the full name of the wave, because it represents the wave structure of radial, azimuth, and axial waves-spaces. The axial wave, represented by the function (4.34), propagates along $Z$-axis in the positive direction. The convergent radial wave $\Psi_{1 / 2}^{-}$corresponds to the divergent one,

$$
\begin{equation*}
\Psi_{1 / 2}^{-}=A i \frac{e^{i(\omega t+k r)}}{\sqrt{k r}} e^{-i\left(1 / 2 \varphi+\varphi_{0}\right)} e^{-i k_{z} z} \tag{4.35}
\end{equation*}
$$

Both waves form the dynamic stationary wave field in the radial direction, expressed mathematically by the standing radial wave:

$$
\begin{equation*}
\Psi_{1 / 2}=\Psi_{1 / 2}^{+}+\Psi_{1 / 2}^{-}=i a \frac{\cos k r \cdot e^{i \omega t}}{\sqrt{k r}} e^{-i\left(1 / 2 \varphi+\varphi_{0}\right)} e^{-i k_{z} z} \tag{4.36}
\end{equation*}
$$

Simultaneously, the $\Psi_{1 / 2}$-wave is the traveling wave in the azimuth and axial directions, positive with respect to the Z-axis:

$$
\begin{equation*}
\left(\Psi_{1 / 2}\right)^{+}=i a \frac{\cos k r}{\sqrt{k r}} e^{-i\left(1 / 2 \varphi+\varphi_{0}\right)} e^{i\left(\omega t-k_{z} z\right)} . \tag{4.37}
\end{equation*}
$$

The corresponding wave, traveling in the negative direction, is

$$
\begin{equation*}
\left(\Psi_{1 / 2}\right)=i a \frac{\cos k r}{\sqrt{k r}} e^{i\left(1 / 2 \varphi+\varphi_{0}\right)} e^{i\left(\omega t+k_{z} z\right)} \tag{4.37a}
\end{equation*}
$$

Both waves form the standing wave in the radial and axial directions:

$$
\begin{equation*}
\Psi_{1 / 2}=i a \frac{\cos k r}{\sqrt{k r}} e^{i\left(1 / 2 \varphi+\varphi_{0}\right)} \cos k_{z} z \cdot e^{i \omega t} \tag{4.38}
\end{equation*}
$$

However, in the azimuth direction, it is the traveling wave along the electron orbit. If we are uninterested in the description of the axial wave, we can omit the axial component and to consider only the radial-azimuth subspace:

$$
\begin{equation*}
\Psi_{1 / 2}=i a \frac{\cos k r}{\sqrt{k r}} e^{i\left(1 / 2 \varphi+\varphi_{0}\right)} \cdot e^{i \omega t} . \tag{4.39}
\end{equation*}
$$

Returning back to the traveling wave (4.34), let us write it as

$$
\begin{equation*}
\Psi_{1 / 2}^{+}=A i \frac{e^{-i\left(k r+k_{z} z+1 / 2 \varphi+\varphi_{0}\right)} e^{i \omega t}}{\sqrt{k r}} . \tag{4.40}
\end{equation*}
$$

A surface of the equal phase of the traveling wave is defined by the equality

$$
\begin{equation*}
\phi=k r+k_{z} z+1 / 2 \varphi+\varphi_{0}=\text { const } . \tag{4.41}
\end{equation*}
$$

If $r=$ const, surfaces of the equal phase represent by themselves the spiral lines with the parameters:

$$
\begin{equation*}
k_{z} \frac{d z}{d t}=-\frac{1}{2} \frac{d \varphi}{d t}=-\frac{\omega_{e}}{2}=-\frac{2 \pi}{2 T_{e}}=-\frac{2 \pi}{T_{\varphi}}=-\omega_{\varphi}, \tag{4.42}
\end{equation*}
$$

where $\omega_{e}$ and $T_{e}$ are the circular frequency and period of the electron's orbiting, $\omega_{\varphi}$ is the circular frequency of the azimuth wave of the fundamental tone and $T_{\varphi}=2 T_{e}$ is its period. Thus, we have

$$
\begin{equation*}
k_{z} v_{z}=-\omega_{\varphi}=-k_{\varphi} v_{\varphi} \quad \text { and } \quad T_{z}=T_{\varphi}, \tag{4.42a}
\end{equation*}
$$

where $v_{z}$ and $v_{\varphi}$ are, correspondingly, the axial and azimuth wave speeds. If the azimuth speed is negative, the clockwise motion occurs and the axial speed is directed in the positive direction along $Z$-axis.

At that, the wave radius $\lambda_{z}$ defines the axial wavelength:

$$
\begin{equation*}
\lambda_{z}=2 \pi \lambda_{z}=-v_{z} T_{\varphi} . \tag{4.43}
\end{equation*}
$$

Such a wave should be called, strictly speaking, the azimuth axial wave because it is determined by the azimuth period. Because the azimuth axial wave (4.43) is related to two orbital turns, it is equal to the double wave screw pitch.

Under $z=$ const, surfaces of the constant phase (4.41) are spiral and the following equality is valid:

$$
\begin{equation*}
k \frac{d r}{d t}=-\omega_{\varphi} . \tag{4.44}
\end{equation*}
$$

At the cosmic levels, these are represented by the spiral galaxies, and in the microworld, they express the same wave structure.

When $r=$ const and $z=$ const, the constant phase of the spatial-time wave $\phi=\omega t-k r-k_{z} z-1 / 2 \varphi-\varphi_{0}=$ const , at the circular orbit, is connected with the frequency of the fundamental tone by the equality: $\omega d t-1 / 2 d \varphi=0$. From this we arrive at

$$
\begin{equation*}
\omega=\omega_{\varphi} \tag{4.45}
\end{equation*}
$$

During the one wave period of the fundamental tone, the electron twice runs the azimuth orbit, therefore, $\Delta \varphi=\frac{2 \pi}{l}=4 \pi$, where $l=m=1 / 2$ is the order of the function and $m$ is the azimuth number. When the order of the wave function is equal to $l=m=p$, the constant phase of the azimuth wave, at $r=$ const and $z=$ const , is represented by the equality

$$
\begin{equation*}
\phi=\omega t-k r-k_{z} z-p \varphi-\varphi_{0}=\text { const } . \tag{4.46}
\end{equation*}
$$

Then, during the one wave period, the electron's azimuth displacement will be of

$$
\begin{equation*}
\Delta \varphi=2 \pi / p \tag{4.47}
\end{equation*}
$$

The amplitude of $\Psi$-function of displacement, in the cylindrical electron wave, obviously, takes the following form

$$
\begin{equation*}
A_{m}=A / \sqrt{\rho} . \tag{4.48}
\end{equation*}
$$

In such a case, the amplitude speed of the wave will be equal to

$$
\begin{equation*}
v_{m}=\omega A_{m}=\frac{\omega A}{\sqrt{\rho}} . \tag{4.49}
\end{equation*}
$$

Because the circular motion is the sum of two mutually perpendicular po-tential-kinetic waves, hence, like the case the string (see Eq. (2.61)), the amplitude energy of an orbiting electron is

$$
\begin{equation*}
E=m v_{m}^{2}=m \omega^{2} A_{m}^{2}=\frac{m \omega^{2} A^{2}}{k r}=\frac{m \omega^{2} A^{2}}{\frac{\omega}{v_{0}} r}=\frac{m A^{2} v_{0}}{r} \omega=\hbar_{e} \omega . \tag{4.50}
\end{equation*}
$$

As we see, the electron energy is proportional to the circular frequency, i.e., to the time wave number of the time field of the orbit, $\omega$. We will rewrite its as

$$
\begin{equation*}
E=m v_{m}^{2}=\hbar_{e} \omega=h_{e} v=h_{e} \frac{v_{0}}{\lambda_{e}}, \tag{4.51}
\end{equation*}
$$

where $\lambda_{e}$ is the electron wave of $H$-atom space,

$$
\begin{equation*}
\hbar_{e}=\frac{m A^{2} v_{0}}{r} \quad \text { and } \quad h_{e}=\frac{2 \pi m A^{2} v_{0}}{r} \tag{4.51a}
\end{equation*}
$$

are the orbital, radial and azimuth, electron actions. In the space of the stationary field of standing waves, we have the same (4.51) relations, however, here

$$
\begin{equation*}
\hbar_{e}=\frac{m a^{2} v_{0}}{r} \quad \text { and } \quad h_{e}=\frac{2 \pi m a^{2} v_{0}}{r} . \tag{4.52}
\end{equation*}
$$

On the other hand, the electron is in the spherical field of $H$-atom, where its action is the constant value. Hence, at $r=r_{0}, a=r_{0}$ (see Eq. (4.25a)) and we have

$$
\begin{equation*}
\hbar_{e}=m v_{0} a \tag{4.53}
\end{equation*}
$$

The wave atomic space, with the wave frequency $\omega=v_{0} / \lambda_{e}$, induces outside the atomic space the external waves of matter-space-time of the same frequency, but with the speed $c$ and wavelength $\lambda$, so that

$$
\begin{equation*}
\omega=\frac{v_{0}}{\lambda_{e}}=\frac{c}{\lambda} . \tag{4.5}
\end{equation*}
$$

Therefore, the electron energy can be presented also as (see Eq. (4.23a))

$$
\begin{equation*}
E=\hbar_{e} \omega=h_{e} \nu=h_{e} \frac{c}{\lambda}=\frac{1}{2} h_{\lambda} \frac{c}{\lambda} . \tag{4.55}
\end{equation*}
$$

With that, the electron's wave energy is equal its kinetic energy on the orbit:

$$
\begin{equation*}
E=\hbar_{e} \omega=m v_{0} r_{0} \omega=m v r \omega=\frac{1}{2} m v r \omega_{\text {orb }}=\frac{1}{2} m v^{2}, \tag{4.55a}
\end{equation*}
$$

where $\omega=(1 / 2) \omega_{\text {orb }}$ is the circular wave frequency of the fundamental tone and $\omega_{\text {orb }}$ is the circular frequency of electron's revolution along the orbit, for which $v=r \omega_{o r b}$.

Consequently, the energies $E=h v$ and the above-considered actions $h$, has no relation to the mystic massless quanta of Einstein's pure energy (which move in "vacuum" with light speed and have the wave features). These are the fundamental parameters of the electron orbit and electron itself and, hence, of the $\mathbf{H}$-atom on the whole.

How can a mystic photon have physical properties, if its expansion (size), at least in the direction of the propagation of the wave beam, is equal to zero (according to the "great relativity theory")? The mystic photon is an object being outside of real space. It is a fruit of a headlong fantasy of abstractionism and machism, the fig-leaf covering the mechanical model of the wave field (proposed by Einstein) and nothing more.

Now, we will turn to the $\Psi$-function of the electron. Let us assume that the electron orbit is in the plane $z=0$. Because the electron is the node of the wave orbit, hence, the boundary orbital conditions at the instant $t=0$ must express the equality to zero of potential azimuth displacements in the node during one revolution:

$$
\begin{equation*}
\left.\operatorname{Re} e^{-i\left(1 / 2 \varphi+\varphi_{0}\right)}\right|_{\varphi=0}=\left.\operatorname{Re} e^{-i\left(1 / 2 \varphi+\varphi_{0}\right)}\right|_{\varphi=2 \pi}=0 \tag{4.56}
\end{equation*}
$$

These conditions are realized for the traveling electron wave in the positive direction if, e.g., $\varphi_{0}=\pi / 2$. In such a case, $\Psi$-function of the electron takes the
form:

$$
\begin{align*}
& \Psi_{1 / 2}^{+}=A i \frac{e^{i(\omega t-k r)}}{\sqrt{k r}} e^{-i(1 / 2 \varphi+\pi / 2)} e^{-i k_{z} z}  \tag{4.57}\\
& \Psi_{1 / 2}=\frac{A}{\sqrt{\rho}} e^{i / 2 / \varphi \varphi} e^{i k_{z} z} e^{i(\omega t-\rho)} . \tag{4.57a}
\end{align*}
$$

he function (4.57) describes the wave of the fundamental tone of the electron $\lambda_{e}$. Its length, as was revealed earlier, is equal to the doubled length of the electron orbit of the Bohr radius $r_{0}$ :

$$
\begin{equation*}
\lambda_{e}=4 \pi r_{0} . \tag{4.58}
\end{equation*}
$$

The wave motion of the fundamental tone occurs in the nearest layers of the wave atmosphere of the H -atom, almost at its surface (we will discuss it later).

The equilibrium wave interchange of energy takes place between the $H$ atom and the surrounding field of matter-space-time. However, under the perturbations, the electron wave (4.58) can reiterate itself in the cosmic wave of the same frequency (see Eq. (4.54):

$$
\begin{equation*}
\lambda_{c}=\frac{4 \pi r_{0}}{v_{0}} c \tag{4.59}
\end{equation*}
$$

The inverse quantity of this wave is the Rydberg constant:

$$
\begin{equation*}
R=\frac{1}{\lambda_{c}}=\frac{v_{0}}{4 \pi r_{0}} \frac{1}{c}=\frac{1}{T_{0} c} . \tag{4.60}
\end{equation*}
$$

This formula has been obtained earlier at an analysis of oscillations of a string and we arrived at it again on the basis of the accurate wave calculation of the H -atom's space.

Let us turn now to the energetic calculation of electron transitions in the $H$ atom. The electron realizes the transitions of the $H$-atom from the $n$-th into $m$-th energetic state; it is the wave motion with the energy of transition (4.55). The law of conservation of energy, at such an extremely fast "quantum" transition, can be presented by the equality:

$$
\begin{equation*}
E_{m}+h \frac{c}{\lambda}=E_{n} \tag{4.61}
\end{equation*}
$$

Taking into account the potential energy of the electron in the spherical field of the $H$-atom (4.26), we obtain the following equation of the energetic balance:

Hence, we have

$$
\begin{gather*}
h \frac{c}{\lambda}=E_{n}-E_{m}=\frac{m v_{0}^{2}}{2}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right)  \tag{4.62}\\
\frac{1}{\lambda}=\frac{m v_{0}^{2}}{2 h c}\left(\frac{1}{m^{2}}-\frac{1}{n^{2}}\right) \tag{4.63}
\end{gather*}
$$

Thus, in the strict correspondence with the wave theory, without any hypotheses and formal fittings, we arrive at the spectral formula of $H$-atom (4.63) and the Rydberg constant

$$
\begin{equation*}
R=\frac{m v_{0}^{2}}{2 h c}=\frac{v_{0}}{4 \pi r_{0} c} \tag{4.64}
\end{equation*}
$$

Note once more, in conclusion, the wave solutions for $l \neq 0$ do not concern the $H$-atom. They describe the shell structure of the others atoms of Mendeleev's periodic table (we will consider it in Chapter 6).

## 5. Hydrogen atom by Schrödinger's equation

Let us remember some facts from history. E. Schrödinger published his "For Lectures on Wave Mechanics" in 1928. Three years later, H.E. White presented cross-sections of polar components of the $\Psi$-function *. Up to that time, scientific works in the decadence spirit were widespread. For example, the Pauli exclusion principle (a result of a free game of notions) was put forward in 1925. A whole series of theoretical constructions was carried out on the basis of the Rutherford-Thomson nuclear model, where the key role was attributed to electrons, etc. In these circumstances, Schrödinger was, to a definite degree, a hostage of the pressure of physical-mathematical decadence, which widely publicized quantum mechanics, as the highest achievement of human thought.

In this connection, it is appropriate to quote some discourses of Joseph Clapper, a specialist on free information **. He has stated that there is the sphere, where the mass media is extremely effective with the opinions of new problems. Under "new problems", he meant the problems about which neither an individual nor his friends (or members of the same group) have no opinion. The reason for the efficiency of the mass media, in the creation of views on new problems, is obvious. An individual is unable to protect oneself and, hence, information moves to unprotected ground. And, as soon as the opinion was formed, it becomes a new opinion, which can be strengthened easily, but changed in a very difficult way. Especially, such a process of formation of a definite view is the most efficient, when a man has no other sources of information. Thus, he depends on the mass media, accessible for him, yet more.

In 1933, Max Born wrote in his book on atomic physics *** (All presented here and further quotations were taken from Russian editions of references): "Primarily, Schrödinger undertook an attempt to regard the corpuscles, in particular electrons, as wave packets. Although Schrödinger's formulae are quite correct, his interpretation could not hold out against the critique. Actually, on the one hand, the wave packets run throughout the course of time. However, on the other hand, the description of the interaction between two electrons, as the interaction of two wave packets in the ordinary three-dimensional space, encounters enormous difficulties.

The current accepted interpretation was proposed by Born. From the points of this interpretation, the probabilistic laws define a whole course of events in a physical system. A probability, defined by de Broglie's wave (associated with the state of a particle), corresponds to a location of a particle in space. Thus, the mechanical process is conjugated with the wave process, i.e., with the propaga-

[^4]tion of a probabilistic wave. This wave, submitted to Schrödinger's equation, defines the probability of any variant of course of events in the mechanical frame of references. If for example, a wave of probability has the zero amplitude, in a point of space, it means that the probability to find an electron in this point is disappearingly small" (page 117).
urther, on pages 172-173, we find: "In conclusion, we should consider once again the sense of the wave function itself... It should be expected that in wave mechanics as well, the wave function $\psi$, or rather its modulus squared, must gain in importance, because an instant value itself of the oscillating function, of course, cannot play a role because of the high frequency of oscillations. The reason for taking the square of the modulus is that the wave function itself (because of the imaginary coefficient of the time derivative in the differential equation) is a complex quantity, while quantities susceptible of physical interpretation must of course be real". (Underlined by the authors)

These judgements call up Newton's and Euler's views. L. Euler, in his "Algebra" (1770), has asserted: "Square roots of negative numbers are not equal to zero, are not less than zero, and are not greater than zero. From this it is clear that the square roots of negative numbers cannot be among the possible (actual, real) numbers. Hence, we have no another way except to acknowledge these numbers as impossible ones. This leads us to the notion of numbers, impossible in essence, which are usually called imaginary (fictitious) numbers, because they exist only in our imagination."

In connection to the statistical interpretation of the wave function, M. Born has continued (pages 174-175): "We speak often about the distribution of electron density in the atom or about the electron cloud around the nucleus. We mean by these words the charge distribution, which is obtained under the multiplication of the function of probability $\left|\psi_{n}\right|^{2}$ by the electron charge $e$. According to the statistical interpretation, the sense of this distribution is clear: in the photos of Fig. 16, it is shown how this distribution can depict. The image represents projections (shadows) of electron clouds in different states."
A.K. Timiryazev gave the critique estimation to quantum mechanical concepts, which are based on positivism. His book * contains a critical analysis of the status quo in physics at that time (which has the actual meaning so far), concerning the acceptation of the statistical approach to the description of the atomic structure. Below are some passages from it.
"Beginning from 1923, the Bohr theory experienced deep crisis. Great difficulties have appeared during attempts to extend the theory ... for other atoms... Further, vast obstacles have arisen when it appeared necessary to calculate time, during the course of which the electron radiates energy, jumping from one orbit to another.

[^5]This crisis was "resolved" in 1925, when Heisenberg's paper on the abstract mathematical theory of quanta appeared, where any hints of a model were deliberately removed... Thus, the new theory represents by itself a step back....

It is very characteristic that neither Heisenberg nor Schrödinger... deny the fact of the existence of atoms and electrons. But because of the fear of difficulties, which they encountered in the Bohr theory, they rejected ideas about motion of electrons in the atom. In new theory, there is nothing about an orbit of an electron...
... Let us cite... the characteristic of Heisenberg's theory given by H . Thirring (1888-1976, Austrian-born physicist) *: "Heisenberg's theory represents a naked calculation scheme... The nakedness and unobviousness of this scheme are deliberate and desired. Responding to the question about the appearance of an atom, Heisenberg could have answered approximately in the following way: 'fool, your question is senseless like the question of a child: was the infant Christ a boy or a girl?'. An atom is a thing, which should not look like something, in such a degree, similar to... who has signs of the sex and nationality. Atomic features are exhibited for our world of senses through the spectra, energetic levels, which are found under the excitation of atoms. And these quantities are uniquely defined by the laws of quantum mechanics".
...It is interesting to note that Professor Sommerfeld, assuming that new theory of quanta represents a great achievement, nevertheless, very unambiguously speaks about its dark sides... Sommerfeld's views are so interesting that we cite their here completely ${ }^{* *}$ : "The clearly expressed intention, which was in the first Heisenberg work on quantum mechanics, is the development of the method, which will be based exceptionally on the relations between the observable values in principle. Such notions as, for example, the location of an electron, the time of its revolution, and the form of the path of its motion, must be rejected from consideration. This limitation, in framework of only principally observable quantities, is based after all on Mach's philosophy. 30 years ago, leaning directly upon Mach, the popularization of the so-called energetics took place. Energetics has proposed to recognize only values of energy in the capacity of physically observed quantities".

Lois de Broglie wrote, in the Preface to D. Bohm's book ***: "Classical physics has allowed the description of the evolution of events in nature, as causally unfolded in space and time $\ldots$ and, consequently, the creation of the clear and exact models (for the imagination of a physicist). On the contrary, modern quantum physics forbids any presentation of such kind and makes it impossible. It accepts only such theories, whose foundation is made up of quite abstract for-

[^6]mulae, and cast doubts on an idea related to the causal run of atomic and corpuscular phenomena. It recognizes only the laws of probability and regards these laws as the laws, having the primary character and forming the final cognizable reality. It does not permit explaining these laws as a result of a causal evolution that occurs at the deeper level of the physical world...".

Lois de Broglie also noted in this work that physicists-theorists "under the influence of preconceived ideas, originated from the positivistic doctrine, assume..., that the indefinite and incomplete character of knowledge about the real processes in physics of the microworld (obtained from the experiment at its modern stage) is a result of the real indeterminacy of physical states and their changes. Hence, such an interpretation, evidently, is not a justified one. Perhaps, in the future, during the process of the cognition of the deeper level of physical reality, we will be able to interpret the probabilistic laws and quantum mechanics as a statistic result of the manifestation of the entire determined values of variables, which, in our time, are hidden".

In the conclusion of the Preface, Lois de Broglie wrote: Professor Bohm showed "that quantum mechanics has no right to regard its own modern views as final. And it is unable to forbid explorers to suppose an existence of the domains of reality, which are deeper than those, already investigated".

Note one more important work by L.V. de Broglie *. A manuscript of this work, published in 1982, is dated of 1950-1951, i.e., Lois de Broglie decided to publish it only 30 years later. In Preface, "The evolution of Lois de Broglie's ideas concerning interpretation of wave mechanics", de Georges Lochak (an editor of a French edition) focused attention on de Broglie's doubts, concerning the right understanding of quantum mechanics bases:
"His doubts became apparent through a great number of remarks, glued in between the manuscript pages, ... corrections, and small but essential additions, which are in the book... A great deal changed during the 30 years, but at that time, an unexpected turn (change) of the views of one of the prominent physicists of the 20th century was sensational and in some degree, even scandalous. In the corridors of Henri Poincaré's Institute, all have spoken about Lois de Broglie in a low voice, as if a terminal disease had suddenly been discovered and as if it should reasonable be aside from him... The debate has grown to a scale of religious wars and have obtained the world character."

After careful research of many problems related to the atomic theories, "he came to the conclusion that the difficulties of the nuclear theory, like the difficulties of quantum electrodynamics, are unremovable in the framework of generally accepted ideas. These difficulties caused by the fundamental incapacity of all theory, in the large, to describe the space-time structures. In his

[^7]stronger growing persuasion, fully developed to 1952, he expressed by the following unequivocal phrase: 'On this day, all possibilities of explanation of phenomena by wave mechanics, in that form, as it teaches, are, in a considerable extent, exhausted'."

Such are the historical estimations of quantum mechanics, as a theory, born on the basis of different philosophical trends of subjective character. Through these trends, one can refer to the different currents of the "philosophy of the blind". The formal mathematical guessing and fitting to the experiment, in the spirit of abstractionism, is the essence of this philosophy. Organs of eyesight and, hence, the right cerebral hemisphere of man are announced as unnecessary ones, even harmful. The left cerebral hemisphere and a free game of notions of learned blinds is accepted as praiseworthy. It is not necessity to prove that a blind man, from his birth, is not able to understand the World, like quantum mechanics, which represent by itself the theory of aggressively blind theorists. At the beginning of the 21st century, it is time to recognize this impartial truth.

It should also be noted that all humankind experience teaches that with the motion "up", in Cosmos, it increases indeterminacy. On the contrary, with the motion "down", in the microworld, it increases definiteness and accuracy, which are verified by modern electron technologies.

After general historical and philosophical remarks, let us turn now to the analysis of Schrödinger's radial functions, which are not of a suit with wave fields. We will begin from $s$-states of $H$-atom.

Schrödinger's radial functions of $s$-states define the electron density of probability, increasing the origin of coordinates. Hence, such solutions, unconditionally, point to the instability of the proton-electron dynamic system, more exactly, the impossibility of an existence of such a system.

About this disagreeable effect of Schrödinger's equation, N. Mott and I. Sneddon have gingerly noted as far back as the year, 1948 *. They have written that "wave spherical functions of $s$-states are spherically symmetric ones and they have the maximum in the origin of coordinates, at $r=0$. That means that probability of the penetration of the electron in an internal domain of the atom in such a case is too big" (taken from the Russian version of the reference *).

Two physicists-theorists, Hans A. Bethe and Edwin E. Salpeter, have expressed quite opposite assertions **: "... We present the dependence $r^{2} R_{n, l}^{2}(r)$ on $r$, i.e., the probability that the electron will be found in a spherical layer, between radii $r$ and $r+d r$. ... It is clear that the maximal charge density accounts

[^8]for the bigger and bigger radii, according to the increase of the main quantum number $n$ ". Let us discuss the above-quoted polar statements.

Schrödinger's radial functions for $H$-atom are determined by the expression

$$
R_{n, l}=N_{n, l} e^{-\rho /(\kappa+1+l)}\left(\frac{2 \rho}{\kappa+1+l}\right)^{l}\left(a_{0}+a_{1}\left(\frac{2 \rho}{\kappa+1+l}\right)+\ldots+a_{\kappa}\left(\frac{2 \rho}{\kappa+1+l}\right)^{\kappa}\right),(5.1)
$$

where $n=\kappa+l+1$ is the main quantum number, $a_{i}$ are constant coefficients of Laguerre polynomial.

All radial functions of $s$-states, for which $l=0$, have the form

$$
\begin{equation*}
R_{n, 0}=N_{n, 0} e^{-\rho /(\kappa+1)}\left(a_{0}+a_{1}\left(\frac{2 \rho}{\kappa+1}\right)+\ldots+a_{\kappa}\left(\frac{2 \rho}{\kappa+1}\right)^{\kappa}\right) . \tag{5.2}
\end{equation*}
$$

The first extremum of the radial functions of $s$-states falls on the origin of coordinates and is equal to

$$
\begin{equation*}
\left(R_{n, 0}\right)_{\max }=a_{0} N_{n, 0} . \tag{5.3}
\end{equation*}
$$

It means that the most probable place of localization of the electron is in the center of nucleus. Then, it is not clear, how the electron pierces the nucleus, whose density (according to the nuclear model) is about $2.5 \cdot 10^{14} \mathrm{~g} / \mathrm{cm}^{3}$. It is impossible to imagine such a density. Most likely, something is wrong with the nuclear atomic model.

Thus, for $s$-states, the density of probability is turned out to be maximal, in the origin of coordinates, and extremely small, on the orbit. So that, the electron must leave the orbit and, penetrating the nucleus, be in its center. How the electron contrives to penetrate the nucleus, of unimaginably huge density, is a baffling question.

Hence, the main effect of solutions of the equetion (5.1) is that $s$-states destroy the Bohr postulate on the stability of orbiting of the electron and show that the electron, certainly, will fall dawn on the nucleus and, what is more, permeate into its central part.

Here is a simple example. Under the condition $\kappa=0$ and $l=0$, an infinite series of the radial function is cut off at the zero member and the radial function $R_{n, l}(\rho)$ takes the form

$$
\begin{equation*}
R_{1,0}=2 e^{-\rho} . \tag{5.4}
\end{equation*}
$$

Its root, $\rho_{1,0, \text { max } 1}=0$, corresponds to the maximum of this function which results in the infinite value of energy:

$$
\begin{equation*}
W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0}(0)^{2}}=-\infty . \tag{5.5}
\end{equation*}
$$

The absurdity is evident that does not require comments.
After getting rid of such disagreeable conclusions, the new wave radial function $P_{n, l}=R_{n, l} r$ was invented, frankly in the spirit of a free game of no-
tions. How did they do it? Very simply. The real radial function $R_{n, l}$ was arbitrary multiplied by the radius $r$. Such a mathematical game allowed to "prove" the stability of the $H$-atom "in full agreement with quantum mechanics". Obviously, $P_{n, l}=R_{n, l} r=0$ is always at $r=0$.

In accordance with Born's interpretation, the density of probability to find the electron in an arbitrary point of space is defined by the modulus squared of Schrödinger's wave function. In spherical polar coordinates, the wave function has the form

$$
\begin{equation*}
\Psi=\psi_{n, l}(\rho, \theta, \varphi) e^{i \omega t}, \tag{5.6}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi_{n, l}(\rho, \theta, \varphi)=R_{n, l}(\rho) \Theta_{l, m}(\theta) \Phi_{m}(\varphi) \tag{5.6a}
\end{equation*}
$$

is the amplitude (spatial) function.
The probability density of location of the electron in an elementary volume $d V$ is determined by the differential

$$
\begin{equation*}
d w=\psi_{n, l} \psi_{n, l}^{*} d V=\rho_{r} \rho_{\theta} \rho_{\varphi} d V=\rho_{w} d V \tag{5.7}
\end{equation*}
$$

where $\rho_{r}=R_{n, l}^{2}, \rho_{\theta}=\Theta_{l, m}^{2}$, and $\rho_{\varphi}=\left|\Phi_{m}\right|^{2}=\frac{1}{2 \pi}$ are, correspondingly, the radial, polar, and azimuth components of the probability density; $\rho_{w}=\rho_{r} \rho_{\theta} \rho_{\varphi}$ is the total hypothetical probability density, which can be presented as

$$
\begin{equation*}
\rho_{w}=\psi_{n, l} \psi^{*}{ }_{n, l}=\frac{1}{2 \pi} R_{n, l}^{2} \Theta_{l, m}^{2} . \tag{5.8}
\end{equation*}
$$

As follows from the equation (5.7), dimensionality of the wave function $\Psi$ (if the length of space is measured in centimeters) is

$$
\begin{equation*}
\operatorname{dim} \Psi_{n, l}=\operatorname{dim} \psi_{n, l}=1 / \mathrm{cm}^{3 / 2} \tag{5.9}
\end{equation*}
$$

This dimensionality has no physical sense, because no physicist can show in reality the space with the dimensionality $\mathrm{cm}^{3 / 2}$. In this sense, Schrödinger's wave function is a mathematical fantasy, far from reality. However, this drawback can be easily removed if the density of probability is determined according to the following expression:

$$
\begin{equation*}
\rho_{w}=\frac{1}{\Omega_{0}} \psi_{n, l} \psi_{n, l}^{*}=\frac{1}{2 \pi \Omega_{0}} R_{n, l}^{2} \Theta_{l, m}^{2}, \tag{5.10}
\end{equation*}
$$

where $\Omega_{0}$ is an elementary volume of space, which is determined on the basis of the wave equation, or some physical conditions. From Schrödinger's equation it follows that

$$
\begin{equation*}
\Omega_{0}=a^{3}=\left(\frac{r_{0}}{Z}\right)^{3}, \tag{5.11}
\end{equation*}
$$

where $r_{0}$ is Bohr radius, Z is the ordinal number of a hydrogen-like atom.

A hypothetical picture of the most probable electron density is defined by the extremes of the total density of probability in the three-dimensional space. But, because the azimuth component is constant, hence, the most probable distribution is determined, in essence, by two extremes of densities, $R_{n, l}^{2}$ and $\Theta_{l, m}^{2}$.

In spherical polar coordinates, an elementary volume $d V=r^{2} \sin \theta d \varphi d \theta d r$ allows writing an elementary probability (which represents, strictly speaking, the third order differential) as

$$
\begin{equation*}
d^{3} w=\frac{1}{a^{3}}\left(\frac{1}{\sqrt{2 \pi}} R_{n, l} \Theta_{l, m}\right)^{2} r^{2} \sin \theta d \varphi d \theta d r=\left(\frac{1}{\sqrt{2 \pi}} R_{n, l} \Theta_{l, m}\right)^{2} \rho^{2} \sin \theta d \varphi d \theta d \rho, \tag{5.12}
\end{equation*}
$$

where $\rho=r / a$.
If we require the consideration of the probability within a spherical layer of an interval $(\rho, \rho+d \rho)$, then it is natural to perform the partial integration with respect to $\varphi$ and $\theta$. It leads to the second order differential

$$
\begin{equation*}
d w=\left(\int_{0}^{2 \pi}\left|\Phi_{m}\right|^{2} d \varphi \int_{0}^{\pi} \Theta^{2} \sin \theta d \theta\right) R_{n, l}^{2} \rho^{2} d \rho \tag{5.13}
\end{equation*}
$$

The coefficients of polar and azimuth functions are determined, in quantum mechanics, on the basis of the integrals of normalization of these functions:

$$
\begin{equation*}
\int_{0}^{2 \pi}\left|\Phi_{m}\right|^{2} d \varphi=1, \quad \int_{0}^{\pi} \Theta_{l m}^{2} \sin \theta d \theta=1 \tag{5.14}
\end{equation*}
$$

In virtue of this, the equation (5.13) can be rewritten in the following form

$$
\begin{equation*}
d w=R_{n, l}^{2} \rho^{2} d \rho \tag{5.15}
\end{equation*}
$$

The coefficient of the radial function is defined by the integral

$$
\begin{equation*}
\int_{0}^{\infty} R_{n, l}^{2} \rho^{2} d \rho=1 \tag{5.16}
\end{equation*}
$$

The equation (5.16) allows the manipulation with the notion of the radial density of probability by introducing the new radial function $P_{n, l}=R_{n, l} \rho$ and, correspondingly, the pseudo density - the "radial probability":

$$
\begin{equation*}
D_{n, l}=P_{n, l}^{2}=R_{n, l}^{2} \rho^{2} \tag{5.17}
\end{equation*}
$$

The pseudo density $D$ creates an illusion as if the probability of location of the electron in the origin of coordinates, i.e., in the center of $H$-atom, is equal to zero. Here is an example (taken from the Russian version of the book *) of technology of formation of the radial component of pseudo $\Psi$-function:

[^9]"The most probable value $r$ is obtained from the expression $P(\mathbf{r}, t)=\Psi^{*}(\mathbf{r}, t) \Psi(\mathbf{r}, t)$. The probability of location of the electron within a small domain between $r$ and $r+\Delta r$ is equal, at $\Delta r \rightarrow 0$, to
$$
P_{n, l}(r) d r=\lim _{\Delta r \rightarrow 0}\left[\int_{0}^{2 \pi \pi} \int_{0}^{r+\Delta r} \int_{r}^{r} \psi_{n, l, m}^{*} \psi_{n, l, m} r^{2} d r \sin \theta d \theta d \varphi\right]=\eta_{n, l}^{2}(r) r^{2} d r,
$$
where $P_{n, l}(r)$ is the function of the radial distribution for a hydrogen-like atom (the probability calculated per unit length).... Returning to the expression for the radial distribution of probability density $P_{n, l}(r)$, we can write
$$
P_{n, l}(r)=\left[r \eta_{n, l}(r)\right]\left[r \eta_{n, l}(r)\right]=\left[R_{n, l}(r)\right]^{2},
$$
where $R_{n, l}(r)$ is determined as the radial amplitude function".
Analyzing the above-presented material, we observe an excellent game of symbols in the required direction. (For definiteness, let us designate the pseudo radial function by the symbol $\widetilde{R}_{n, l}(r)$ ). At first, the standard symbol of the radial function $R_{n, l}(r)$ is substituted for the nonstandard symbol $\eta_{n, l}(r)$. Then, using the standard symbol of the radial function $R_{n, l}(r)$, the pseudo radial function is introduced in accordance with the equality
\[

$$
\begin{equation*}
R_{n, l}(r)=r \eta_{n, l}(r) . \tag{5.18}
\end{equation*}
$$

\]

Such a logical tightrope walking means only one thing: "the radial function does not suit us because it shows that the orbiting electron is unstable; this is why we substitute the symbol, and the notion itself, of the radial function $R_{n, l}(r)$ according to the following correspondence:

$$
\begin{equation*}
R_{n, l}(r) \quad \rightarrow \quad \widetilde{R}_{n, l}(r)=r R_{n, l}(r) " . \tag{5.18a}
\end{equation*}
$$

After such manipulations, there is no choice, as only to present the diagrams of the "radial amplitude function $R_{n, l}(r)$ ", i.e., the pseudo radial function $\widetilde{R}_{n, l}(r)$. But now we have the absolutely other picture of the distribution, where the "probability" to find an electron in the center of a nucleus, in all cases, will be equal to zero.

In actual fact, if we will lean upon the real radial function, we must recognize that at $l=0$ motion of the electron around the nucleus is, to the higher degree, the unstable one and the electron must "bore" the nucleus and prove to be in its center. However, such a conclusion was undesirable for creators of quantum mechanics.

Thus, removing the troublesome radial function $R_{n, l}(r)$ (or that is the same $\left.R_{n, l}(\rho)\right)$ from the probabilistic description of the electron motion and substituting it for the pseudo radial function $r R_{n, l}(r)$ (or $\rho R_{n, l}(\rho)$ ), the "explanation" of
an existence of stationary states of the hydrogen atom, in accordance with Bohr theory, "turns out well" in some cases.

Of course, W.Flygare had no relation to the birth of such a mathematical fitting. He plainly tried to substantiate by his own way the evident substitution of Schrödinger's radial function for the pseudo radial function.

On the basis of formulae (1.9), (1.9a), and (5.1), Schrödinger's $\Psi$-function for $H$-atom can be written as

$$
\begin{align*}
\Psi_{n, l}= & \frac{N_{n, l}}{2 \pi} e^{-k r /(\kappa+1+l)}\left(\frac{2 k r}{\kappa+1+l}\right)^{l}\left(a_{0}+a_{1}\left(\frac{2 k r}{\kappa+1+l}\right)+\ldots\right.  \tag{5.19}\\
& \left.\ldots+a_{\kappa}\left(\frac{2 k r}{\kappa+1+l}\right)^{\kappa}\right) \Theta_{l, m}(\theta) e^{-i m \varphi} e^{i \omega t}
\end{align*}
$$

where $k=1 / a$ is the number, inverse to Bohr radius $a$.
Let the coefficient of the variable $r$ will be presented in the following form:

$$
\begin{equation*}
k_{r}=\frac{i k}{\sigma+l} \tag{5.19a}
\end{equation*}
$$

where $\sigma=\kappa+1$ is the number of members of Laguerre polynomial. Then the Laguerre polynomial can be written as

$$
\begin{equation*}
f\left(-2 k_{r} i r\right)=\left(a_{0}+a_{1}\left(\frac{2 k r}{\kappa+1+l}\right)+\ldots+a_{\kappa}\left(\frac{2 k r}{\kappa+1+l}\right)^{\kappa}\right) . \tag{5.19b}
\end{equation*}
$$

As a result, Schrödinger's $\Psi$-function will be presented in the following form

$$
\begin{equation*}
\Psi_{n, l}=\frac{N_{n, l}}{2 \pi} f\left(-2 k_{r} i r\right) e^{i\left(\omega t+k_{r} r\right)} \Theta_{l, m}(\theta) e^{-i m \varphi} \tag{5.20}
\end{equation*}
$$

The radial component of the wave function

$$
\begin{equation*}
R_{n, l}=N_{n, l} f\left(-2 k_{r} i r\right) e^{i\left(\omega t+k_{r} r\right)} \tag{5.21}
\end{equation*}
$$

represents by itself the wave with the imaginary radial wave number and the amplitude (5.19b). At that the frequency $\omega$ is also the "imaginary" one:

$$
\begin{equation*}
\omega=k_{r} v_{0}=\frac{i k v_{0}}{\sigma+l}, \tag{5.21a}
\end{equation*}
$$

where $V_{0}$ is the wave speed.
Since the wave number $k_{r}$ of Schrödinger's space is an imaginary quantity, therefore, Schrödinger's $\Psi$-functions do not describe the spatial waves; they express the aperiodic perturbations of space independently of an interpretation of $\Psi$-function.

Accordingly, because of this reason as well, solutions of Schrödinger's equation do not correspond to the real wave picture of the microworld. They exclude the formation of waves of $H$-atom in the surrounding space that is in full contradiction with reality.

As was already mentioned in Section 1 of this chapter, according to Born's interpretation, the extremes of radial functions $R_{n, l}$ define the radii of shells of the most probable states: $r=a \rho_{n, l, \text { max } i}$, where $i$ is the number of an extremum. However, in an overwhelming number of cases, relative values of radii of extremal shells are not equal to integers squared: $\rho_{n, l, \text { max } i} \neq m^{2}$ and, hence, these negate the condition of cutting off (1.5). Such shells define the energetic levels, non-existent in nature:

$$
\begin{equation*}
W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a \rho_{n, l, \max i}} . \tag{5.22}
\end{equation*}
$$

Of all pseudo functions, only pseudo functions of the kind $(R \rho)_{n, n-1}=$ const $\exp (-\rho / n) \cdot \rho^{n-1}$ have extremes corresponding to the integers squared: $\rho=n^{2}$; hence, they "confirm" Bohr's orbits: $r=a_{0} n^{2}$. In this connection, let us consider in detail the radial and pseudo radial solutions. We will denote the radii of shells of extremes of radial and pseudo radial functions, correspondingly, as $\rho_{n, l, \max i}$ and $\rho_{n, l, \max i s}$.

The following pseudo radial function $P_{1,0}$ corresponds to the radial function $R_{1,0}=2 e^{-\rho}:$

$$
\begin{equation*}
P_{1,0}=2 e^{-\rho} \rho . \tag{5.23}
\end{equation*}
$$

Its root is $\rho_{1,0, \max 1 s}=1$. The energy, corresponding to it,

$$
W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0}(1)^{2}},
$$

satisfies $H$-atom spectrum. However, the pseudo function itself (5.23) is not a solution of Schrödinger's equation. So that, we have no right to apply its roots to the formula (5.22), which is related to the other (radial) function.

The rest of radial and pseudo radial functions, for $n=2,3,4,5$, have the form as follows.
$n=2$,

$$
W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 2^{2}}(\text { Energy by Bohr })
$$

By Schrödinger:
$l=0, \kappa=1$ : $R_{2,0}=\frac{1}{2 \sqrt{2}} e^{-\rho / 2}(2-\rho):$
$\rho_{2,0, \max 1}=0, \quad \rho_{2,0, \max 2}=2^{2} \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0}(0)^{2}}=\infty, \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 2^{2}}$.
Although the last energy satisfies the cutting off condition, but simultaneously the first root negates it.

$$
\begin{aligned}
& P_{2,0}=\frac{1}{2 \sqrt{2}} e^{-\rho / 2}(2-\rho) \rho \text { (The pseudo radial function): } \\
& \rho_{2,0, \max , s}=0.7639320225 \quad 5.236067977499 \\
& \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{2,0, \max , s}} \quad \rightarrow \quad R=\infty, \\
& l=1, \kappa=0: \quad \quad R_{2,1}=\frac{1}{2 \sqrt{6}} e^{-\rho / 2} \rho: \\
& \rho_{2,1, \max 1}=2 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 2} \quad \rightarrow \quad R=\infty . \\
& P_{2,1}=\frac{1}{2 \sqrt{6}} e^{-\rho / 2} \rho^{2} \text { (The pseudo radial function): } \\
& \rho_{2,1, \max 1 s}=4 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 2^{2}} \text {. The formal "conformity" with the Bohr }
\end{aligned}
$$ energy takes place here (the fitting happens). In this case ( $l=1, \kappa=0$ ), the maximum density of the hypothetical probability $R_{2,1}$ is defined by the root $\rho_{2,1, \max 1}=2$, which eliminates the cutting off condition and, hence, the value of the radial function becomes infinite, $R=\infty$. This unpleasantness has been "removed" by means of the pseudo radial function $P_{2,1}$ the maximum of which $\rho_{2,1, \max 1 s}=2^{2}$ gives the energy, corresponding formally to the Bohr energy of the electron. But, as was elucidated above, this root has no relation to the radial function and, correspondingly, energy (5.22). This absurdity is inherent for all other functions.

$\underline{n=3}, \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 3^{2}}$ (Energy by Bohr)
By Schrödinger:

$$
\begin{array}{ll}
l=0, \kappa=2: & R_{3,0}=\frac{2}{81 \sqrt{3}} e^{-\rho / 3}\left(27-18 \rho+2 \rho^{2}\right): \\
\rho_{3,0, \max 1}=0 & \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{3,0, \max 1}}=\infty \\
\rho_{3,0, \max 2,3}=3.531370333, & 11.46862697 ; W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{3,0, \max }} \quad \rightarrow \quad R=\infty, \\
P_{3,0}=\frac{2}{81 \sqrt{3}} e^{-\rho / 3}\left(27-18 \rho+2 \rho^{2}\right) \rho \text { (The pseudo radial function): } \\
\rho_{3,0, \text { max }, s}=0.7400370804 & 4.185930091
\end{array}
$$

$$
\rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{3,0, \max , s}} \quad \rightarrow \quad R=\infty
$$

$$
l=1, \kappa=1:
$$

$$
\begin{gathered}
R_{3,1}=\frac{4}{81 \sqrt{6}} e^{-\rho / 3} \rho(6-\rho), \\
\rho_{3,1, \max }=1.75735931 \quad 10.242640687 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{3,1, \max }} \quad \rightarrow R=\infty .
\end{gathered}
$$

$$
P_{3,1}=\frac{4}{81 \sqrt{6}} e^{-\rho / 3} \rho^{2}(6-\rho) \text { (The pseudo radial function): }
$$

$$
\rho_{3,1, \max , s}=2.227998127 \quad 10.7720018726 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{3,1, \max , s}} \quad \rightarrow R=\infty .
$$

$$
l=2, \kappa=0:
$$

$$
R_{3,2}=\frac{4}{81 \sqrt{30}} e^{-\frac{\rho}{3}} \rho^{2}:
$$

$$
\rho_{3,2, \max 1}=6 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 6} \quad \rightarrow \quad R=\infty
$$

$$
\begin{aligned}
& P_{3,2}=\frac{4}{81 \sqrt{30}} e^{-\frac{\rho}{3}} \rho^{3} \text { (The pseudo radial function): } \\
& \rho_{3,2, \max 1 s}=9 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 3^{2}} \text {. The formal"conformity" with }
\end{aligned}
$$

Bohr energy takes place - the fitting happens.
$\underline{n=4}, \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 4^{2}}$ (Energy by Bohr)
By Schrödinger:
$l=0, \kappa=3$ :

$$
\begin{gathered}
R_{4,0}=\frac{1}{768} e^{-\rho / 4}\left(192-144 \rho+24 \rho^{2}-\rho^{3}\right): \\
\rho_{4,0, \max 1}=0 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 0^{2}}=\infty, \\
\rho_{4,0, \max 2-4}=3.419589235 \quad 9.90409441 \quad 22.673316353 \\
W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,0, \max 2-4}} \rightarrow \quad R=\infty . \\
P_{4,0}=\frac{1}{768} e^{-\rho / 4}\left(192-144 \rho+24 \rho^{2}-\rho^{3}\right) \rho \text { (The pseudo radial function): } \\
\rho_{4,0, \max 1 s}=0.732343789 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,0, \max 1 s}} \quad \rightarrow \quad R=\infty,
\end{gathered}
$$

$$
\rho_{4,0, \max 2 s}=4 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 2^{2}} \quad \rightarrow \quad R=\infty
$$

(The last energy corresponds to the $H$-atom spectrum, but it does not satisfy the cut-off condition by the reason which were elucidated above and leads to the divergence of the radial function, $R=\infty$ )

$$
\begin{aligned}
& \rho_{4,0, \max 3,4 s}=10.649564176 \quad 24.618092034 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,0, \max 3,4 s}} \rightarrow R=\infty . \\
& ----------------1=1,
\end{aligned}
$$

$$
\begin{gathered}
R_{4,1}=\frac{1}{256 \sqrt{15}} e^{-\rho / 4} \rho\left(80-20 \rho+\rho^{2}\right): \\
\rho_{4,1, \text { max }}=1.69690610,8.7491349,21.55395899 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,1, \text { max }}} \rightarrow R=\infty . \\
P_{4,1}=\frac{1}{256 \sqrt{15}} e^{-\rho / 4} \rho^{2}\left(80-20 \rho+\rho^{2}\right) \text { (The pseudo radial function): } \\
\rho_{4,1, \text { max }, s}=2.8303936591 \quad 9.589180277 \quad 23.580426063 \\
\rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,1, \text { max }, s}} \quad \rightarrow \quad R=\infty .
\end{gathered}
$$

$l=2, \kappa=1:$

$$
\begin{gathered}
R_{4,2}=\frac{1}{768 \sqrt{5}} e^{-\frac{\rho}{4}} \rho^{2}(12-\rho): \\
\rho_{4,2, \max 1,2}=5.071796769 \quad 18.928203230 \quad \rightarrow W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,2, \max 1,2}} \rightarrow R=\infty .
\end{gathered}
$$

$$
P_{4,2}=\frac{1}{768 \sqrt{5}} e^{-\rho / 4} \rho^{3}(12-\rho) \text { (The pseudo radial function): }
$$

$$
\rho_{4,2, \max , s}=6.788897449 \quad 21.2111025509 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,2, \max , s}} \quad \rightarrow \quad R=\infty .
$$

$$
l=3, \kappa=1
$$

$$
\begin{aligned}
& R_{4,3}=\frac{1}{768 \sqrt{35}} e^{-\rho / 4} \rho^{3}: \\
& \rho_{4,3, \max 1}=12 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{4,3, \max 1}} \quad \rightarrow \quad R=\infty . \\
& P_{4,3}=\frac{1}{768 \sqrt{35}} e^{-\frac{\rho}{4}} \rho^{4} \text { (The pseudo radial function): } \\
& \rho_{4,3, \max 15}=16 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 4^{2}} .
\end{aligned}
$$

The formal "conformity" with Bohr energy takes place here - the fitting has happened)
$n=5$,
$W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} 5^{2}}($ Energy by Bohr $)$

By Schrödinger:

$$
\begin{gathered}
l=0, \kappa=4: \quad R_{5,0}=\frac{2}{46875 \sqrt{5}} e^{-\frac{\rho}{5}}\left(9375-7500 \rho+1500 \rho^{2}-100 \rho^{3}+2 \rho^{4}\right): \\
\rho_{5,0, \max }=0 \quad 3.3730382 \quad 9.46156165 \quad 19.4953468 \quad 37.6700533 \\
P_{5,0}=\frac{2}{46875 \sqrt{5}} e^{-\rho / 5}\left(9375-7500 \rho+1500 \rho^{2}-100 \rho^{3}+2 \rho^{4}\right) \rho: \\
\rho_{5,0, \max , s}=0.7288877086 \\
39.9231294250
\end{gathered}
$$

$$
l=1, \kappa=3:
$$

$$
P_{5,1}=\frac{4}{46875 \sqrt{30}} e^{-\frac{\rho}{5}} \rho^{2}\left(3750-1125 \rho+90 \rho^{2}-2 \rho^{3}\right):
$$

$$
\begin{array}{llll}
\rho_{5,1, \max , s}=2.7638777872 & 9.0359270606 & 19.3000556408 & 38.9001395111
\end{array}
$$

$$
l=2 \quad \kappa=2:
$$

$$
\begin{aligned}
& R_{5,2}=\frac{4}{46875 \sqrt{70}} e^{-\frac{\rho}{5}} \rho^{2}\left(525-70 \rho+2 \rho^{2}\right): \\
& \rho_{5,2, \max }=4.804896151 \quad 15.9559233 \quad 34.2391805
\end{aligned}
$$

$$
P_{5,2}=\frac{4}{46875 \sqrt{70}} e^{-\frac{\rho}{5}} \rho^{3}\left(525-70 \rho+2 \rho^{2}\right):
$$

$$
\rho_{5,2, \max , s}=6.3157051741 \quad 16.990494945 \quad 36.693799880
$$

$$
l=3, \kappa=1:
$$

$$
R_{5,3}=\frac{4}{46875 \sqrt{70}} e^{-\rho / 5} \rho^{3}(20-\rho): \quad \rho_{5,3, \max 1}=10 \quad \rho_{5,3, \max 2}=30
$$

$$
P_{5,3}=\frac{4}{46875 \sqrt{70}} e^{-\rho / 5} \rho^{4}(20-\rho): \quad \rho_{5,3, \max 1,2 s}=12.192235935 \quad 32.807764064
$$

$$
l=4, \kappa=0:
$$

$$
R_{5,4}=\frac{4}{140625 \sqrt{70}} e^{-\rho / 5} \rho^{4}, \quad \rho_{5,4, \max 1}=20 \quad \rightarrow \quad W=-\frac{e^{2}}{8 \pi \varepsilon_{0} a_{0} \rho_{5,4, \max 1}} \rightarrow R=\infty
$$

$$
P_{5,4}=\frac{4}{140625 \sqrt{70}} e^{-\rho / 5} \rho^{5}, \quad \rho_{5,4, \max 1 s}=5^{2} \quad \rightarrow \quad W=-\frac{Z^{2} e^{2}}{8 \pi \varepsilon_{0} a_{0} 5^{2}}
$$

(The formal conformity with Bohr energy takes place here - the fitting happens)
Let us once more analyze the radial and pseudo radial functions.
An introduction of the pseudo radial function means the substitution of Schrödinger's $\psi_{n, l}$-function (5.19) for the new (pseudo) function $\Xi_{n, l}=r \cdot \psi_{n, l}$, which does not satisfy Schrödinger's wave equation,

$$
\begin{equation*}
\Xi_{n, l}=r \cdot \frac{N_{n, l}}{2 \pi} e^{i\left(\omega t-k_{n} r\right)} f\left(2 k_{n} r\right) \Theta_{l, m}(\theta) e^{-i m \varphi}, \tag{5.24}
\end{equation*}
$$

where $k_{n}=\frac{k}{k+l+1}$. At that the tacit transformation of the three-dimensional space of events into the one-dimensional space took place. Now, the H -atom is represented by a linear formation.

Disregarding all above-stated and the fact that $\Xi$-function is not a solution of Schrödinger's equation, let us continue the discussion: "is an introduction of this function, as the probabilistic function, rightful? The modulus squared, $\Xi_{n, I} \Xi_{n, l}^{*}$, defines the density of "probability" (compare with (5.8)):

$$
\begin{equation*}
\rho_{w}=\Xi_{l, m} \Xi_{l, m}^{*}=\frac{1}{2 \pi} r^{2} R_{n, l}^{2} \Theta_{l, m}^{2}=\frac{1}{2 \pi} P_{n, l}^{2} \Theta_{l, m}^{2} . \tag{5.24a}
\end{equation*}
$$

Everything is revealed through a comparison; therefore, we will consider the classical problem, where we will deliberately make the same error, like the one accomplished at the introduction of the radial pseudo function $P_{n, l}$.

Let us assume that some material points-events are randomly distributed in three-dimensional space, with the increasing density to the origin of coordinates, according to Gaussian distribution:

$$
\begin{equation*}
\rho_{w}=\frac{1}{\sqrt{(2 \pi)^{3}} \cdot \sigma^{3}} e^{-\frac{r^{2}}{2 \sigma^{2}}}, \tag{5.25}
\end{equation*}
$$

where $\sigma$ is some parameter of distribution.
The probability, that the material points-events are located within a small domain, between $r$ and $r+\Delta r$, is

$$
\begin{equation*}
\Delta w=\int_{r}^{r+\Delta r} \rho_{w} d V=\int_{r}^{r+\Delta r} \rho_{w} 4 \pi r^{2} d r=\int_{r}^{r+\Delta r} \frac{1}{\sqrt{(2 \pi)^{3}} \cdot \sigma^{3}} e^{-\frac{r^{2}}{2 \sigma^{2}}} 4 \pi r^{2} d r . \tag{5.26}
\end{equation*}
$$

If $\Delta r \ll 1$, then the integral (5.26) can be substituted for the finite differential

$$
\begin{equation*}
\Delta w=\rho_{w} \Delta V=\frac{1}{\sqrt{(2 \pi)^{3}} \cdot \sigma^{3}} e^{-\frac{r^{2}}{2 \sigma^{2}}} 4 \pi r^{2} \Delta r \tag{5.26a}
\end{equation*}
$$

where $\Delta V=4 \pi r^{2} \Delta r$ is an elementary volume of space, presented in the form of a spherical layer of the thickness $\Delta r$.

For the comparison of probabilities of events in different elementary domains of three-dimensional space, we must take only equal volumes $\Delta V$. Otherwise, the picture of probability will be distorted.

Probabilities of events in two arbitrary equal spherical layers-volumes,

$$
\begin{aligned}
& \Delta w\left(r_{1}\right)=\frac{1}{\sqrt{(2 \pi)^{3}} \cdot \sigma^{3}} e^{-\frac{r_{1}^{2}}{2 \sigma^{2}} 4 \pi r_{1}^{2} \Delta r_{1},} \\
& \Delta w\left(r_{2}\right)=\frac{1}{\sqrt{(2 \pi)^{3}} \cdot \sigma^{3}} e^{-\frac{r_{2}^{2}}{2 \sigma^{2}} 4 \pi r_{2}^{2} \Delta r_{2},}
\end{aligned}
$$

are defined by the following relation

$$
\begin{equation*}
\frac{\Delta w\left(r_{1}\right)}{\Delta w\left(r_{2}\right)}=e^{-\frac{r_{1}^{2}-r_{2}^{2}}{2 \sigma^{2}}}=\frac{\rho_{w}\left(r_{1}\right)}{\rho_{w}\left(r_{2}\right)} . \tag{5.26b}
\end{equation*}
$$

The partition of the space of events into equal volumes, in which, directly or indirectly, frequencies of random events are counted up, is an indispensable condition. It is the indispensable procedure for the probabilistic estimations of events. Accordingly, it is inadmissible to violate this axiom if we wish to see the objective picture of events, but not its distorted image.

For example, we are interested in the probabilistic trajectory of the movement of a Mr. "N", within the bounds of a small town. For this purpose, we must divide the town into parts of equal areas. Then, it is necessary to count up the frequency of appearances of " N " there (in every part of the town) during a long enough period of time. As a result, we will obtain the field of probability of the events. Obviously, in this field of events, some line of extremal values of probability, which defines the probabilistic movements of " N ", will be found. The probabilistic trajectory will coincide, mainly, with the route to his work, and it also will reflect his definite business and private interests.

Taking into consideration all of the aforementioned, we can return to our problems. What will happen if the formula of differential probability (5.26a) will be rewritten as follows?

$$
\begin{equation*}
d w=\rho_{w} d V=\left(\frac{4 \pi r^{2}}{\sqrt{(2 \pi)^{3}} \cdot \sigma^{3}} e^{-\frac{r^{2}}{2 \sigma^{2}}}\right) d r \tag{5.27}
\end{equation*}
$$

Following formal logic, the change of positions of cofactors in a formula does not change its product; so that, no errors were committed at rewriting of the equation (5.26a) in the form of the equation (5.27). Dialectical logic asserts that at the permutation of cofactors (which resulted in the formation of new cofactors) there is no quantitative error, but a chance exists for qualitative error. The latter leads to gross distortions of the described events and to the erroneous quantitative effects.

In the expression (5.27), the first step, on the way to distortions of the objective field of probability, was made. Namely, the differential volume $d V=4 \pi r^{2} d r$ was substituted for the new differential $d r$, which takes the constant value in all points of the space of events. In such a case, the former equal differentials $d V$ will be related between themselves as

$$
\begin{equation*}
\frac{d V_{2}}{d V_{1}}=\frac{r_{2}^{2}}{r_{1}^{2}} \tag{5.28}
\end{equation*}
$$

Now, it is impossible to speak about a comparison of events. The density of pseudo probability can be called the "radial probability" or the "linear density". It must be written, according to the equation (5.27), in the following form

$$
\begin{equation*}
\rho_{r}=4 \pi r^{2} \rho_{w}=\frac{4 \pi r^{2}}{\sqrt{(2 \pi)^{3}} \cdot \sigma^{3}} e^{-\frac{r^{2}}{2 \sigma^{2}}} \tag{5.29}
\end{equation*}
$$

For comparison, the plots of functions of the density $\rho_{w}$ and the pseudo density $\rho_{r}$, for $\sigma=1$, are presented in Fig. 3.8.


Fig. 3.8. Plots of the density $\rho_{w}$ and the pseudo density $\rho_{r}$.

The graph of the real density of probability $\rho_{w}$, in the three-dimensional space, shows that the probability of distribution of material points-events increases in the direction of the origin of coordinates. Quite the contrary, the pseudo probability, related to the linear space (where there are no material points-events) points out that the "probability" of finding the events falls to zero in the origin of coordinates.

It is a direct evidence of absurdity. Figuratively saying, operating with mathematics in the spirit of a free game of notions, the "black" (the maximal distribution in the origin of coordinates) is announced as the "white" (the zero distribution in the origin of coordinates). This absurdity has happened because the equal differential volumes of space, $d V=4 \pi r^{2} d r$, were deliberately substituted for the differentials-functions

$$
\begin{equation*}
d V(r)=4 \pi r^{2} d(\text { const }) \tag{5.30}
\end{equation*}
$$

where $d$ (const) $=d r$ is the constant differential in all space of events.

As an example of inadmissibility of such erroneous transformations, let us imagine a spherical galaxy, in which the density of stars is expressed by the function (5.25). The above mentioned manipulation with the linear density will lead to the laughable conclusion, as if the probability of finding of stars falls to zero, according as we approach to the center of the galaxy.

We can supplement the problem, in question, with an example of the twodimensional distribution of material points-events by the Gaussian distribution with the density of probability

$$
\begin{equation*}
\rho_{w}=\frac{1}{\sqrt{(2 \pi)^{2}} \cdot \sigma^{2}} e^{-\frac{r^{2}}{2 \sigma^{2}}} \tag{5.31}
\end{equation*}
$$

and the differential probability

$$
\begin{equation*}
d w=\rho_{w} d S=\frac{1}{\sqrt{(2 \pi)^{2}} \cdot \sigma^{2}} e^{-\frac{r^{2}}{2 \sigma^{2}}} 2 \pi r d r \tag{5.32}
\end{equation*}
$$

Here, the falsification of events is also possible through an introduction of the "radial probability"

$$
\begin{equation*}
\rho_{r}=\frac{2 \pi r}{\sqrt{(2 \pi)^{2}} \cdot \sigma^{2}} e^{-\frac{r^{2}}{2 \sigma^{2}}}=r e^{-\frac{r^{2}}{2 \sigma^{2}}} \tag{5.33}
\end{equation*}
$$

which announces the zero probability in the origin of coordinates.
Let us imagine an ant-hill in a forest. Its external contours correspond to the graph of the density of probability (5.31) (Fig. 3.9a).


Fig. 3.9. The transformation of the structure of a studying object (a), described by the density of probability $\rho_{w}$, similar to (5.31), into the completely other object (b), described by the different, in principle, "radial probability" $\rho_{r}$ (5.33).

Obviously, the probability of distribution of ants in the ant-hill follows the similar formula of density. Now, we decided to prove that, as approaching to the center of the ant-hill, the probability of finding of ants there falls. To this end, it is necessary to take advantage of the "radial probability" (5.33) (Fig. 3.9b). We propose to theorists of such probabilities to poke a hand in the center of an anthill in order to examine the correctness of their "theory".

Thus, in positivism, the notion the truth is a subjective notion. The true one is that it satisfies "practical needs" of a theory. And the correspondence of a theory to the real picture of nature has, most often, the speculative character, because the world, by positivism, is the subjective reality.

For the completeness of the above analysis, we should touch the case of the linear distribution of probability of material points-events by Gauss, with the density of probability

$$
\begin{equation*}
\rho_{w}=\frac{1}{\sqrt{2 \pi} \cdot \sigma} e^{-\frac{r^{2}}{2 \sigma^{2}}} \tag{5.34}
\end{equation*}
$$

and the differential probability

$$
\begin{equation*}
d w=\rho_{w} d r=\frac{1}{\sqrt{2 \pi} \cdot \sigma} e^{-\frac{r^{2}}{2 \sigma^{2}}} d r . \tag{5.35}
\end{equation*}
$$

In this situation, the actual probability and the "radial probability" are equal (coincide); accordingly any distortions are impossible here.

In virtue of all above-stated, an introduction of the pseudo radial function in the three-dimensional space of events of the H -atom is unlawful and represents the method of the coarse fitting to the required results. Such formal constructions obtained the wide dissemination in physics.

Let us consider now the discrepancy of the radii of stationary shells, defined by Schrödinger's radial functions, with the radii, defined by Bohr postulates. As an example, we will take the radial function $R_{3,0}$, corresponding to the numbers $\kappa=2$ and $l=0(n=3):$

$$
\begin{equation*}
R_{3,0}=\frac{2}{81 \sqrt{3}} e^{-\frac{\rho}{3}}\left(27-18 \rho+2 \rho^{2}\right) . \tag{5.35}
\end{equation*}
$$

One- (a) and two-dimensional (b) graphs of change of the radial component of density of probability $R_{3,0}^{2}$, in the dependence on the distance, along the radius $\rho$, are presented in Fig. 3.10.


Fig. 3.10. The density of hypothetical probability of $s$-state, $R_{3,0}^{2}$, for Schrödinger's $\Psi$-function with the parameters, $n=3$ and $l=0$.

The radial function squared, $R_{3,0}^{2}$, has the maximum in the origin of coordinates. There are also two smaller maxima, defining the two shells of the most probable localization of the electron (if we will strictly follow the quantum mechanical interpretation of $\Psi$-function). Extremes of the radial function are as follows:

$$
\rho_{3,0, \max 1}=0, \quad \rho_{3,0, \max 2}=3.531370333, \quad \rho_{3,0, \max 3}=11.46862697
$$

At the same time, according to the cut-off condition, only the radius $\rho=3^{2}=9$ defines the stationary shell of the electron, corresponding to this function. Two vertical lines in Fig 3.10a, at the distance equal to 9 from the origin of coordinates, indicate its location. As we see, there is no maximum (shell) of such radius among extremes of $R_{3,0}^{2}$ ! It is no wonder, the radial function $R_{3,0}$ is "ignorant". It does not "know" that it represents by itself the reduced function (obtained as a result of the cut off operation) and, therefore, it cannot defines anything here, including the most probable localization of the electron.

Let us assume that the radial solutions for the $H$-atom in quantum mechanics are correct. Then, it is necessary to pay attention to the next myth, concerning an absence of trajectories (orbits) in the solutions of Schrödinger's equation. It is easily to verify that Schrödinger's $\Psi$-functions define orbits, along which (in accordance with his equation) electrons move.

The natural wave motion occurs along wave trajectories. For example, the trajectories of charged particles are observed experimentally, but they are called, by the modest word, "tracks" in order to confirm the "absence" of trajectories. Here, we meet with the practical application of the method of "a free game of notions", when the indisputable fact of an existence of trajectories is linguistically concealed. A trace of a charged microparticles, for example in the Wilson cloud chamber, is similar to a trajectory of a jet plane, which left a trace on its way, which can be also called a "track". But such a verbal trick cannot change the reality of an existence of trajectories of planes. An analysis of trajectories of charged particles in the magnetic field lies in the base of mass spectrographs. Why, if they exist outside, cannot trajectories be inside atoms?

Schrödinger's equation was built on the basis of an introduction the energy of an electron (orbiting around a proton) in the wave equation. Namely, it was constructed on the basis of Kepler's laws, which uniquely presume an existence of trajectories. Hence, solutions of Schrödinger's equation must contain the probabilistic trajectories.

The calculations of the three-dimensional distribution of maxima of the modulus of the wave function $\psi_{n, 3,1}$ are presented in Fig. 3.11. These maxima point to an existence of three probabilistic circular trajectories (Fig. 3.11d).

An existence of probabilistic trajectories is quite natural, since Schrödinger's equation realizes the transition from an orbit to the probabilistic
field of motion. Extremes of the wave function in such a field must coincide with these trajectories. The data, presented in Fig. 3.31, confirm this.

In the above-presented example, the modulus squared $\left|\psi_{n, 3,1}\right|^{2}$ defines three circular trajectories: two polar, $C_{n}$ and $C_{s}$, and one equatorial, $C_{0}$. Two polar trajectories are defined by principal extremes of the wave function; the equatorial trajectory - by the collateral extremes.


Fig. 3.11. The polar surface of the modulus $\left|\Theta_{3,1}(\theta)\right|$ (a); the cones of the maximal density of the polar probability (b); the combination of the sphere of the maximal radial probability density and the cones of the maximal polar probability density (c); the three-dimensional distribution of maxima of the modulus of the amplitude function $\left|\psi_{n, 3,1}\right|=\left|R_{n, 3}(\rho) \Theta_{3,1}(\theta) \Phi_{1}(\varphi)\right|$, or $\left|\psi_{n, 3,1}\right|=\left|R_{n, 3}(\rho) \Theta_{3,1}(\theta)\right|$ (d).

On such orbits, as was discussed above, the total energy of an electron does not conform to the hydrogen atom spectrum. It is impossible also to co-ordinate the most probable polar trajectories with the usually accepted, quantum mechanical, interpretation of Schrödinger's equation solutions *.

In the spirit of traditions of some physicists of the first half of the $20^{\text {th }}$ century, M. Clyde and J. Selbin have regarded, in their book **, Schrödinger's equation as a particular case of the general wave equation. It is impossible to disagree with their view:

[^10]"The wave function is presented as $\Psi=\psi(x, y, z) e^{-i o t}$. Then, the general wave equation is transformed, by form, into Schrödinger's wave equation by the following way:
$\nabla^{2} \Psi=\frac{1}{v^{2}} \frac{\partial^{2} \Psi}{\partial t^{2}} \quad \Rightarrow \quad \nabla^{2} \Psi+\frac{\omega^{2}}{v^{2}} \Psi=0$, but because $\quad v=\lambda \omega=\frac{\hbar \omega}{p}=\frac{E}{p}$,
then $\quad \nabla^{2} \Psi+\frac{p^{2}}{\hbar^{2}} \Psi=0 \quad$ and $\quad \Delta \Psi+\frac{2 m E}{\hbar^{2}} \Psi=0, \quad$ or $\quad \Delta \Psi+\frac{2 m(W-U)}{\hbar^{2}} \Psi=0 "$.
Because the wave equation describes the plural mass processes, the value of energy in Schrödinger's equation should be regarded as a fundamental measure of the wave field of the studying object. Hence, there are no bases to assert that the orbits (Fig. 3.11) must belong, without fail, to the electron.
G.C. Pimentel and R.D. Spratley, describing the basic concepts of quantum chemistry *, have noted the following. The standard requirements (to the properties of $\Psi$-function) have "stipulated an appearance, in the wave equation solution, of the linear spectrum, which exactly coincides with the hydrogen atom spectrum" (the Russian version *, p. 30). As follows from the above considered, all these "exact" coincidences is the myth and nothing more.

Another widespread opinion, contained in their work (p. 31), is that " $\psi$ does not contain information about a trajectory of motion of an electron. Actually, in the microworld, the notion of the "electron trajectory", in an atom and molecule, loses a sense. Instead of the trajectory, the modulus squared, $|\psi|^{2}$, allows one to determine only the probability of location of the electron in the volume. The $|\psi|^{2}$-value is bigger, when the probability of location of the electron is higher, and smaller, when the electron appears seldomly. At the nodal surface, where $\psi$ value is equal to zero, the electron could never be ".

All these assertions, which have been repeated for tens of years, do not correspond to reality (see, once more, the solutions of Schrödinger's equation, presented in Fig. 3.11). Apart from the considered above, the next question arises. How has it happened that the field of all $\Psi$-functions is created by one electron, which is "spread" over all space? Maybe it is the field of possible positions of many electrons, belonging to different $H$-atoms. In the last case, Schrödinger's equation describes some "averaged probabilistic" $H$-atom, but this picture does not agree with $H$-atom spectrum.

Regarding the Bohr postulates, they wrote * (p. 46): "... a fault of the atomic planetary model, able to call headache, is the demand that the motion of a charged electron along a curvilinear trajectory will occur without radiation and a loss of energy. This difficulty does not arise in quantum mechanics, since the

[^11]notion of the "trajectory" is absent there. The wave description of the electron does not give any information about an existence of its trajectory. Therefore, the question about a lost of energy, caused by the radiation, in such a model, generally, does not arise. Moreover, it is impossible to say that a trajectory exists. Quantum mechanics plainly does not accept it. At the transition from the classical equation of motion to the equation of quantum mechanics, a member, corresponding to the kinetic energy, was changed by definite means. We can render these changes in the form of the following order: Break off think about trajectories - we cannot approach the nature and describe the atomic properties by the equation of motion, which contains the notion of the "trajectory". And, actually, we must not regret about it. If there is no trajectory, then the electron must not emit energy. As we know, the latter is really so".

As follows from the solution, shown in Fig. 3.11, all above cited quotations are, in essence, emotional assertions. They remind us the literary stories, which do not rely on the solutions of Schrödinger's equation, but only repeat the mythology of formal suppositions. They are not supported by proofs, because, throughout the existence of quantum mechanics, the three-dimensional distribution of extremes of Schrödinger's $\Psi$-functions has never been presented. If only this would be done, a myth on the absence of orbits (trajectories) would disappear.

Finally, let us briefly touch upon the radiation of potential-kinetic waves emitted by the $H$-atom. The natural motion in the Universe is the longitudinaltransverse motion-rest, the main form of which is the spiral-like motion. This motion represents by itself a superposition of two mutually perpendicular planepolarized waves. It is the natural vortical motion-rest. And all natural motions in nature are based on the dynamic equilibrium of exchange of matter-space-motion-rest, which forms the "zero" equilibrium radiation.

If some disturbance of the equilibrium happens, then the wave perturbations of matter-space arise. In particular, the orbital motion of an electron, as the natural motion, takes place arbitrarily for a long time, until the dynamic equilibrium of the exchange of matter-space-motion-rest of the $H$-atom with the ambient field of matter-space, will not be upset.

When we set a charged macroobject in a uniform rotation, the definite perturbation in the surrounding space arises as a result. Such a motion is unnatural (artificial) and, therefore, accompanied always with the wave perturbation.

On the contrary, the orbital motion of an electron is the natural motion in $H$ atom space. It is a part of the wave motion of matter-space at the subatomic level with its own dynamic equilibrium "zero" radiation. Such a state of motion of the electron is continued so long, until some wave perturbations will transfer it in a new stationary state. And again, the electron transition, in the new state, is accompanied with the wave perturbation of the ambient field of matter-space, which raises over the zero exchange (radiation).


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[^1]:    * Waller M.D., Cladni Plates, London, Staples Press, 1960.

[^2]:    * Ch. Bru, Esthćtique de L'abstraction, Paris, 1959, p.3.

[^3]:    * Dictionnaire de la abstraite, Paris, 1957, pp. 2-3.
    ** L.G. Kreidik and G.P. Shpenkov, Alternative Picture of the World, V.2, Bydgoszcz, 1996.

[^4]:    * H.E. White, Phys. Rev., 37, 1416, 1931.
    ** Modern Communication and Foreign Policy, Washington, 1967, pp. 60-61.
    *** Max Born, Atomic Physics, Blackie and Son Limited, London-Glasgow, seventh edition, 1963; Mir, Moscow, 1965.

[^5]:    * A.K. Timiryazev, Introduction in Theoretical Physics, State Publishing House, GTTI, Moscow-1933-Leningrad, pp. 334-336.

[^6]:    * Die Grundgedanken der neueren Quantentheorie, H. Thirring, Ergebnisse der exakten Naturwissensschaften, VII Band, 1928, S. 410.
    ** A. Sommerfeld, Atombau und Spektrallinien. Wellenmechanischer Ergänzungsband, 1929, 44. *** David Bohm, Causality and Chance in Modern Physics, L., 1957; IL, Moscow, 1959 (in Russian.

[^7]:    * L. de Broglie, Les Incertitudes D'Heisenberg et L'Interprétation Probabiliste de la Mécanique Ondulatoire, Gauthier-Villars, Bordas, Paris, 1982. (Russian edition: L. de Broglie, Heisenberg Uncertainty Principle and Probabilistic Interpretation of Wave Mechanics, Mir, Moscow, 1986)

[^8]:    * N. Mott, I. Sneddon, Wave Mechanics and its Application, Oxford at the Clarendon Press, 1948; Russian translation by Nauka, Moscow, 1966, p. 75.
    ** Hans A. Bethe, Edwin E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms, Springer-Verlag, Berlin-Göttingen-Heidelberg, 1957; Russian translation by Fizmatgiz, Moscow, 1960.

[^9]:    * W.H. Flygare, Molecular Structure and Dynamics, Prentice-Hall, Inc., Englewood Cliffts, New Jersey, 1978; Russian translation by Mir, Moscow, 1982, V.1, pp. 180-181.

[^10]:    * L.G. Kreidik and G.P. Shpenkov, Foundations of Physics, Bydgoszcz, 1998.
    ** M. Clyde, Jr., Joel Selbin, Theoretical Inorganic Chemistry, Reinhold Dublishing Corporation, N-Y, Chapman \& Hall, LTD, London; Russian translation by "Himiya", Moscow, 1969, p. 49.

[^11]:    * G.C. Pimentel and R.D. Spratley, Chemical Bounding Clarified Through Quantum Mechanics, Holden-Day Inc., San Francisco-Cambridge-London-Amsterdam, 1970; Russian translation by "Mir", Moscow, 1973.

