Anomalous Magnetic Moment of an Electron, Lamb Shift, and Microwave Background Radiation of Hydrogen Atoms: What Do They Have in Common?

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The wave behavior of the hydrogen atom, as the paired proton-electron system, shows itself in vibrations of the center of mass and an atomic spherical shell. These vibrations superimpose on electron's motion resulted in the magnetic moment anomaly of the electron and in an appearance of the background spectrum which defines, as turned out, Lamb "shifts" of optical spectral lines. The anomalous magnetic moment of the electron is relatively simple calculated on the basis of the approach, which takes into account an influence of the inherent vibrations. The value obtained coincides quite precisely with the experimental data, like in the case of Lamb shifts. Thus, with allowance for the wave behavior of a proton and an electron, we arrive not only at the simple description of the anomalous magnetic moment, but also at the discoveries of the background spectrum of the hydrogen atom and the Lamb shifts nature.

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In full agreement with the *wave-particle* duality, we can consider the hydrogen atom as a wave dynamic formation, namely as the simplest paired centrally symmetric proton-electron wave system. According to the Dynamic Model (DM) [1], which takes into account the wave behavior of particles, a proton, just like an electron or any elementary particle, is a pulsing physical point of space of the spherical structure which is in a state of continuous exchange (interaction) with environment, over the wave spherical shell, at the definite fundamental frequency of pulsations, ω_e. Such particles, being, according to the definition, unceasingly pulsing microobjects, possess internal energy. The value of the latter ($E_0 = m_0 c^2$) is defined by the associated mass of a particle m_0 and the fundamental wave speed c of extension of the pulsations in the surrounding space.

Longitudinal oscillations of the *spherical* wave shell of the proton, at the fundamental frequency ω_e, provide an interaction in radial direction (more correctly *exchange* of matter-space and motion-rest) with the surrounding field-space and with the orbiting electron. The orbital motion of the electron is associated with the transversal *cylindrical* wave field. Therefore, the common three-dimensional wave equation

$$\Delta\hat{\Psi} - \frac{1}{c^2}\frac{\partial^2\hat{\Psi}}{\partial t^2} = 0 . \qquad (1)$$

is valid for both cases. Both dynamic constituents of the proton-electron system have to be described, respectively, by *spherical* and *cylindrical* wave functions [2] that has been realized in the DM.

With the regard for wave processes, the nature of the so-called "anomaly" of the magnetic moment of the electron is explained logically non-contradictory and simple [3]. Let us show this. We will use definitions and parameters characteristic for, and originated from, the DM.

The wave motion of the hydrogen atom, as a paired proton-electron system, generates in the simplest case (in equilibrium) an elementary *electric* (*longitudinal*) *moment* and the corresponding *magnetic* (*transversal*) *moment*. The latter is

$$\mu_e = \frac{\upsilon_0}{c} e(r_0 + \delta r_0) , \qquad (2)$$

where the term δr_0 takes into account all small deviations of the orbital radius r_0 caused by different constituents of specific wave motion of the electron in the intra-atomic wave field; e is the electron's exchange ("electric") charge, υ_0 is the oscillatory speed of boundary wave shell of the hydrogen atom equal to the Bohr speed, c is the base wave (phase) speed of the wave exchange (equal to the speed of light).

The term δr_0 takes into account all additional motions (caused by vibrations) that perturb (modulate) trajectory of the orbiting electron, namely: (1) the circular motion of the center of masses of the hydrogen atom, because the hydrogen atom, as a whole, oscillates in the spherical field of exchange with the amplitude (characteristic for the wave sphere, at $kr = 1$, where k is the wave number) defined by the fundamental wave radius $\lambda\!\!\!^{-}_e$; (2) oscillations of the wave shell together with the orbiting electron and oscillations of the center of mass of the hydrogen atom with the amplitude defined by the Bohr radius r_0 and the first root of the spherical Bessel functions of the zero order $z_{0,s} = b'_{0,1}$ [4] (responding to the extremum of the first kinetic shell [2]); (3) oscillations of the center of mass of the electron itself, as a whole, with respect to the center of mass of the hydrogen atom, defined by the radius of the wave shell of the electron r_e and the roots of Bessel functions responding to zero and maximum of the first kinetic shell, $y_{0,1}$ and $y'_{0,1}$.

The total magnetic moment of the electron $\mu_e(th)$ in an expanded form, followed from the DM, is defined by the equation

$$\mu_e(th) = \frac{e\upsilon_0}{c}\left[r_0 + \left(\frac{c}{\omega_e} + \frac{r_0}{b'_{0,1}}\right)\sqrt{\frac{2Rh}{m_0 c}} + \beta r_e \frac{y_{0,1} + y'_{0,1}}{2 y_{0,1} y'_{0,1}}\sqrt{\frac{2Rh_e}{m_0 c}}\right], \tag{3}$$

where $h_e = 2\pi m_e \upsilon_0 r_e$ is the proper action of the electron (analogous to the Planck action h), m_e is the electron mass. The coefficient $\beta = 1.00155$ takes into account the natural indeterminacy in weight contributions of two items defined by the two roots of Bessel functions, $y_{0,1}$ and $y'_{0,1}$. The substitution of numerical values for all quantities entered in (3) gives the following theoretical values for the total magnetic moment of the electron and its constituents:

$$\begin{aligned}\mu_e(th) &= (1854.801894 + 0.957111963 + 0.112845073 + \\ &+ 0.00550792)\cdot 10^{-26}\ J\cdot T^{-1} = 1855.877359\cdot 10^{-26}\ J\cdot T^{-1}\end{aligned} \tag{4}$$

This magnitude corresponds to Einstein-de-Haas's experimental magnitude, almost coinciding with the latter. The first, major, term in (3) relates to the expected value of the orbital magnetic moment of the electron bound in the hydrogen atom if one supposes that $\delta r_0 = 0$:

$$\mu_{e,orb} = \frac{\upsilon_0}{c} e r_0. \tag{5}$$

A half of this value is called the Bohr magneton:

$$\mu_B = \frac{\upsilon_0}{2c} e r_0 = 927.400947(80)\cdot 10^{-26}\ J\cdot T^{-1}. \tag{6}$$

The latter was introduced in physics due to the erroneous theoretical derivation, that is convincingly shown in [5], of the average value of circular current generated by the orbiting electron. This is why an agreement of Einstein-de-Haas's experimental data with the erroneously derived value μ_e did not happen (two times difference of experimental and theoretical magnitudes took place).

The second and third terms take into account an influence of the aforementioned natural perturbation of the orbital motion of the electron. The only fourth term in (3) has the direct relation to the electron proper (“spin”) magnetic moment, its value (see (4)) is

$$\mu_s = 5.50792\cdot 10^{-29}\ J\cdot T^{-1}. \tag{7}$$

If one subtracts one Bohr magneton μ_B (6) (ascribed erroneously, as was mentioned above, to the electron’s spin magnetic moment) from (4), we obtain the absolute value

$$\mu_e = \mu_e(th) - \mu_B = 928.476412\cdot 10^{-26}\ J\cdot T^{-1}, \tag{8}$$

which precisely coincides with the absolute “2002 CODATA recommended value” accepted for the magnet moment of the electron (within uncertainty in the last two figures):

$$\mu_{e,CODATA} = 928.476412(80)\cdot 10^{-26}\ J\cdot T^{-1}. \tag{9}$$

The magnetic moment of an electron is defined in modern physics by quantum electrodynamics (QED) from the equality

$$\mu_e = (1+a_e)\mu_B, \tag{10}$$

where a_e is called the magnetic moment anomaly of the electron. The latter shows the exceeding of the expected value of the magnetic moment of the electron in one Bohr magneton (6), following from semi-classical field theories.

The whole extended form of the equation on the “anomaly” $a_e(th)$, including functional expressions for factors of the α^n terms, takes many pages. Therefore, we show here only the concise form of the equation derived now [6] up to the forth order in the fine-structure constant [7] α:

$$\begin{aligned} a_e(th) = 0.5\left(\frac{\alpha}{\pi}\right) - 0.328478965579\ldots\left(\frac{\alpha}{\pi}\right)^2 + 1.181241456\ldots\left(\frac{\alpha}{\pi}\right)^3 - \\ -1.5098(384)\left(\frac{\alpha}{\pi}\right)^4 + 4.382(19)\cdot 10^{-12} = 1.1596521535(12)\cdot 10^{-3} \end{aligned}, \tag{11}$$

where

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = 7.297352533\cdot 10^{-3}, \tag{12}$$

$\varepsilon_0 = 8.854187817\ldots\cdot 10^{-12}\ F\cdot m^{-1}$ is the *electric constant*. The derivation of a_e with such a high precision is regarded as one of the advantages of QED.

Let us compare now two presented expressions, (3) and (10), which describe the same quantity – the magnetic moment of the electron. By this way we can compare two theoretical approaches: (1) new one presented here, which takes into account the wave side of nature and can be called therefore *quantum-wave*; and (2) *quantum* (QED).

The derivation by QED (with participation of quantum chromodynamics) of Eq. (10) rests on the concept of virtual particles. The expanded form of the equation is extremely complicated. Actually, the coefficient $1.5098(384)$ of the α^4 term (calculated with big uncertainty, ± 384) consists of more than one hundred huge 10-dimensional integrals. Therefore, because of the complicated mathematical structure of coefficients of the α^n terms, a special system of massively-parallel computers was developed to calculate (10) [6].

Eq. (3), derived on the basis of the quantum-wave approach (realized in the DM [1]), does not contain any integrals, but nevertheless logically and non-contradictory gives the same precise value of μ_e.

Let us turn now to the next two important results originated from the new approach. First, the solution of the wave equation (1) leads also to the spectral formula of the hydrogen atom in an unknown earlier form, where quantum numbers, defining discrete (linear) structure of the optical spectrum, are roots $z_{i,j}$ of Bessel functions $J(kr)$ and $Y(kr)$, *i.e.*, right radial solutions of the wave equation (1) [2, 6]:

$$h\frac{c}{\lambda}=\frac{m_0c^2A^2}{2r_0^2}\left(\frac{\left|\hat{e}_p(kr_m)\right|^2 z_{p,1}^2}{z_{p,m}^2}-\frac{\left|\hat{e}_q(kr_n)\right|^2 z_{q,1}^2}{z_{q,n}^2}\right), \qquad (13)$$

where

$$\hat{e}_l(kr)=\sqrt{\frac{\pi kr}{2}}(J_{l+\frac{1}{2}}(kr)\pm iY_{l+\frac{1}{2}}(kr)), \qquad (14)$$

A is the constant equal to the oscillation amplitude at the sphere of the wave radius ($kr=1$), m_0 and r_0 are mass and radius of the proton.

Second, the natural perturbation of the orbital radius r_0 in the equilibrium state is responsible for the origin of radiation at the level of background. The orbiting electron in hydrogen (both in equilibrium and exited states) *constantly* exchanges the energy with the proton at the fundamental frequency ω_e inherent in the subatomic level. This exchange process between the electron and proton has the dynamically equilibrium character and runs on the background of the superimposed oscillatory field of the center of mass of the proton and its spherical shell.

On the basis of (13), with allowance for the formula on the relative value of the background perturbation $\delta n=\delta r/r_0$ [8, 9] and taking into account the Bessel's functions of the zero order, at $p=q=m=0$, characteristic for the proton-electron system in an equilibrium state, we arrive at the spectrum of the zero wave perturbation, the background spectrum:

$$\frac{1}{\lambda}=R\left(\frac{1}{n^2}-\frac{1}{\left(n+\sqrt{\frac{2Rh}{m_0c}}\cdot\frac{e_p(z_{p,s})}{z_{p,s}}-\beta_n\frac{r_e^2}{r_0^2}\sqrt{\frac{2Rh_e}{m_0c}}\cdot\frac{e_m(z_{m,n})}{z_{m,n}}\right)^2}\right), \qquad (15)$$

where $n=1, 2$; β_n (β_1=1.203068949, β_2=1.018671584) are numerical factors taking into account the fact an excitation of the hydrogen atom on the zero level and using by this reason the first unequal to zero roots of Bessel functions, $j'_{0,2}$ and $j'_{0,3}$, corresponding to the extremes of the first potential radial shells [2]; R is the Rydberg constant; $r_e=4.17052597\cdot10^{-10}\,cm$ is the radius of the *wave* shell of the electron determined in the DM from the formula of mass of elementary particles [1].

The results of calculations by the formula (15) are presented in Tables 1 and 2.

Table 1. The terms, $1/\lambda$, of background spectrum (15) of the hydrogen atom; n = 1.

s	$Z_{p,s}$	$Z_{m,n}$	β_n	$1/\lambda$, cm^{-1} (15)	λ, cm	T, K	$T_{exp,}$, K [10]
1	$y_{0,1}$	$y'_{0,1}$		41.751724	0.023951	12.10805	
2	$y_{0,2}$	$y'_{0,1}$		9.40602023	0.106315	**2.72774**	**2.728 ± 0.002**
	$j'_{0,2}$	$j'_{1/2,1}$	β_1	9.67863723	0.103320	2.80680	
3	$y_{0,3}$	$y'_{0,1}$		5.240486	0.190822	1.51974	
	$j'_{0,3}$	$j'_{1/2,1}$	β_1	5.255841	0.190265	1.52419	

Tablica 2. The terms, 1/λ, of background spectrum (15) of the hydrogen atom; $n = 2$

s	$Z_{p,s}$	$Z_{m,n}$	β_n	1/λ, cm^{-1} (15)	λ, *cm*	T, K
1	$y_{0,1}$	$y'_{0,1}$		5.219748	0.191580	1.5137
2	$y_{0,2}$	$y'_{0,1}$		1.1758681	0.850436	0.3410
	$j'_{0,2}$	$j'_{1/2,1}$	β_2	1.211154	0.825659	0.3512
3	$y_{0,3}$	$y'_{0,1}$		0.6550701	1.526554	0.18997
	$j'_{0,3}$	$j'_{1/2,1}$	β_2	0.6582849	1.519099	0.1909

The zero level of wave exchange (interaction with environment) is integrally characterized by the absolute temperature of zero exchange and exists as a standard energetic medium in the Universe, where hydrogen is the most abundant substance. Actually, the wave $\lambda = 0.106315\,cm$ (see Table 1) is within an extremum of the spectral density of equilibrium cosmic microwave background. The absolute temperature of zero level radiation with this wavelength, which has the *black-body* form [8], is $T = 2.72774\,K$. The *form* of the spectrum of this radiation and its *anisotropy* was *measured* by NASA's COBE satellite ($T_{exp.}$= $2.728 \pm 0.002\,K$ [10]). In 2006 the Nobel Prize was awarded for this work.

An important proof of the correctness of the background radiation formula (15) and, hence, basic features of elementary particles structure, described by the DM, are values of differences of basic energetic terms corresponding to Bessel functions $j'_{0,2}$ and $y_{0,2}$.

It turned out that the theoretical values obtained for the $(j'_{0,2} - y_{0,2})_{n=1}$ (Table 1) and $(j'_{0,2} - y_{0,2})_{n=2}$ (Table 2) terms differences, $\Delta(1/\lambda)cm^{-1}$ [11], coincide with high precision with the most accurate experimental values obtained for the 1*S* and 2*S* Lamb shifts of the hydrogen atom: $L_{1,s} = 8172.837(22)$ MHz and $L_{2s-2p} = 1057.8446(29)$ MHz [12] (Table 3).

Table 3. The frequency gaps, Δν, between the nearest background terms in the hydrogen atom

n	s	Terms differences	Δ(1/λ), cm^{-1}	Δν, MHz	$\Delta\nu_{exp}$, MHz [12]
1	2	$(j'_{0,2} - y_{0,2})_{n=1}$	0.272617	**8172.852**	**8172.837(22)**
	3	$(j'_{0,3} - y_{0,3})_{n=1}$	0.015355	460.3313	
2	2	$(j'_{0,2} - y_{0,2})_{n=2}$	0.0352859	**1057.84466**	**1057.8446(29)**
	3	$(j'_{0,3} - y_{0,3})_{n=2}$	0.0032148	96.37727	

The latter indicates at the natural bond of the Lamb shift with the background spectrum, revealing thus the nature of the “shift” and additionally confirming the correctness of the derived spectrum.

Conclusion

With allowance for the wave behavior of a proton and an electron, we have arrived at the simple, and precise, description of the anomalous magnetic moment of the electron, and also at the discoveries of the background spectrum of the hydrogen atom and the Lamb shifts nature.

Thus, on the basis of the results obtained, we have the right to state that all above phenomena have the same nature of origin conditioned by the peculiarity of the resulting oscillatory-wave motion of the electron. The latter is responsible for the existence of microwave background radiation in the Universe of the pick temperature 2.728 K which we can regard as the zero point energy.

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