

## PHYSICAL MEANING OF IMAGINARY UNIT $i$

George P. Shpenkov

Three centuries have passed since 1712 fierce debates about the meaning of complex numbers were started. Gottfried Leibniz, Leonhard Euler, Johann Bernoulli and other outstanding scientists participated in them. However, from then until all discussions on this topic unfortunately, ended virtually nothing, because they did not lead to the disclosure of the main - the elucidation of the real physical meaning of complex numbers.

We will not go into the history and details of the discussions. We will give exclusively dictums about the nature of complex numbers of some of the well-known scholars whose views do not require any additional comments, so clearly and concisely set out in them the basic idea about the state of the problem.

It is widely known statement about complex numbers belonging to Leibniz (1646-1716):

*"The Spirit of God has found an outlet in subtle analysis of this miracle, freak of the world of ideas, the dual essence, located between being and non-being, which we call the imaginary root of negative unit". Or like this: "Complex numbers are a fine and wonderful refuge of the divine spirit, as if it were an amphibian of existence and non-existence".*

In 1759, Francis Mather (1731-1824) published an article entitled "*Discourse on the use of the algebra of the minus sign*". Here is what he wrote in it about imaginary numbers:

*"As far as I can tell, they serve only to make confusion in the whole theory of equations and make a vague and mysterious in that that by its very nature is particularly clear and simple ... It is highly desirable so to avoid negative roots in algebra, and if they still arise, cast them religiously. There are good reasons to believe that if we could get rid of the negative roots, thus would be removed objections raised by many scholars and men of witty against the algebraic calculations as being too complex and endowed with almost incomprehensible to the mind concepts. Algebra, or universal arithmetic, by their very nature, of course, is the science of not less than a simple, clear and useful for evidence than geometry"*

Leonhard Euler in his fundamental work "*Complete introduction to algebra*" (1770), noting the mysterious unreal nature of imaginary numbers, regarded them as a product of the imagination:

*"Square roots of negative numbers are not equal to zero, are not less than zero, and are not greater than zero. From this it is clear that the square roots of negative numbers cannot be among the possible (actual, real) numbers. Hence, we have no another way except to acknowledge these numbers as impossible ones. This leads us to the notion of numbers, **impossible in essence**, which are usually called **imaginary (fictitious) numbers**, because they exist only in our imagination."*

Not surprising, physicists, faced with complex numbers in their theoretical schemes, perceived imaginary components in them as unrealistic entities (numbers, parameters). So deeply into and fixed in their minds (along with mathematicians), as a matter of course, the unreality of numbers that were called *imaginary*.

Apparently, the mystery of imaginary numbers we have managed to solve, and this report is a brief review of publications on the subject. How did we have come to this discovery, and on which conceptual basis we have based? It will not be difficult to understand, if we look on a sad result with far-reaching consequences to which came physicists because of ignorance of the physical meaning of imaginary numbers and taking the way of mathematical abstractions. As a bright example, it is very instructive to turn to quantum mechanics (QM) [1]. Let us to analyze the very foundations of this theory.

Remember, in order to get rid of the imaginary terms in Schrödinger's complex wave function,

$$\hat{\Psi}_{n,l,m} = R_{n,l}(r)\Theta_{l,m}(\theta)\hat{\Phi}_m(\varphi) = R_{n,l}(r)\hat{Y}_{l,m}(\theta, \varphi), \quad (1)$$

where  $\hat{\Phi}_m(\varphi) = e^{im\varphi}$ , Born in 1926 proposed the well-known probabilistic interpretation of the function. That's how it was done.

Because the “*imaginary*” terms, regarded as unreal quantities, do not have a firm physical explanation, as physicists believed (and believe up to now), he proposed to deal with the square of the modulus of the wave function  $\hat{\Psi}_{n,l,m}$ :

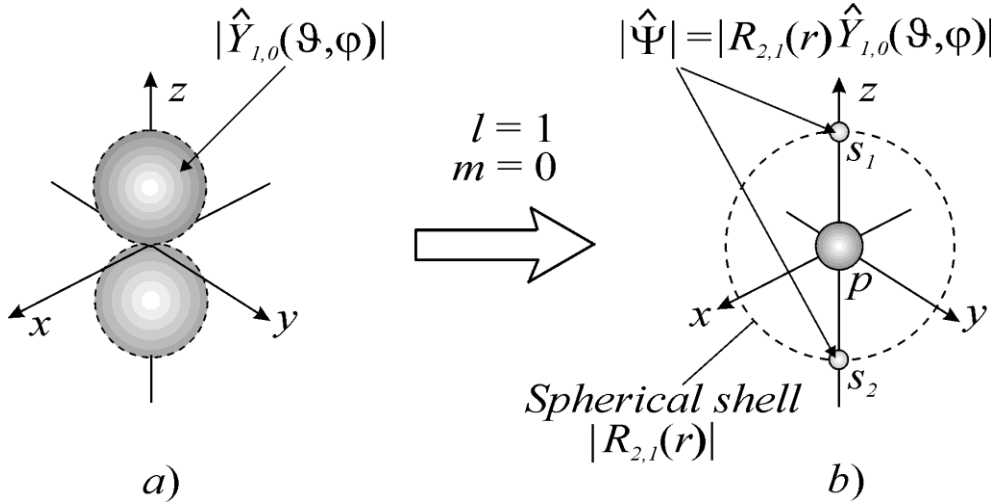
$$\hat{\Psi}_{n,l,m}\hat{\Psi}_{n,l,m}^* = R_{n,l}^2(r)\Theta_{l,m}^2(\theta), \quad (2)$$

to which he ascribed the probability meaning. Here is Born's explanation:

*“The reason for taking the square of the modulus is that the wave function itself (because of the **imaginary** coefficient of the time derivative in the differential equation) is a complex quantity, while quantities susceptible of physical interpretation must of course be real”* [2, p.142].

Following the proposed concept, the probability, for example, of the presence of a single electron in the hydrogen atom at every point and at every instant is proportional to the probability density  $|\hat{\Psi}|^2 = \hat{\Psi}\hat{\Psi}^*$ . This concept on the role of the wave function  $\hat{\Psi}$ , as defining the state of the system through its modulus squared, was taken as the basis in the new theory, QM, and became to be regarded as one of the basic tenets, along with the postulate on the Schrödinger equation and 3 other postulates. Thus was the beginning of quantum mechanics with its probabilistic interpretation and the absence of the concept of the particle's trajectory...

What is the result of the accepted concept? Let us take a look attentively on an example shown schematically in Fig. 1.



**Fig. 1.** Distribution of domains of maxima of the wave function modulus  $|\hat{\Psi}|$  at  $l=1$  and  $m=0$  in a spherical space-field of the hydrogen atom; a)  $|\hat{Y}_{l,m}(\theta, \varphi)|$  is the surface of the modulus of the polar-azimuth factor of the wave function, describes a surface shaped like a dumbbell;  $s_1$  and  $s_2$  (b) are domains of maxima of  $|\hat{\Psi}|$ ;  $R_{n,l}(r)$  is the spherical layer-vicinity corresponding to the maximum of the radial factor of the wave function (a dash circumference);  $p$  is a symbolic image of the nucleus-proton.

Designations presented in the Fig. 1 refer to the distribution of the modulus of the wave function, which is not qualitatively different from that of its square. It is considered a particular case of the solution of the Schrodinger equation corresponding to quantum numbers  $l = 1$  and  $m = 0$ . In this example, extremes of  $|\hat{\Psi}| = |R_{2,1}(r)\hat{Y}_{1,0}(\theta, \varphi)|$  (and, hence,  $|\hat{\Psi}|^2$ ) are in two polar points  $s_1$  and  $s_2$  on the Z-axis, where the spherical surface, determined by the solutions for the radial factor  $R_1(r)$ , and the surface of the “dumbbell” come into contact.

To what led this “solution”? We arrive at the fact that with the equal probability the electron can be exclusively either in  $s_1$  or  $s_2$ . It means that the electron (being in the state determined by the quantum numbers  $l = 1$  and  $m = 0$ ) “hangs” above the “north” or “south” poles of the proton surface, forming together with the proton an electric dipole directed along the polar z-axis. Hence, its orbital (magnetic and mechanical) moments are equal to zero. Obviously, such a structure of the hydrogen atom, originated from the QM interpretation, if we will strictly follow it, is inconsistent with experiment.

The similar inconsistency is inherent in all other functions with different quantum numbers  $l$  and  $m$  [3]. For this reason, throughout an existence of quantum mechanics, seeing this inconsistency and trying not to focus attention at that, a three-dimensional distribution of extremes of Schrödinger's  $\Psi$ -functions has never been presented by physicists-theorists.

Looking at the squaring (2), we see that the result of this action is a removal of the “imaginary” azimuthal function  $\hat{\Phi}_m(\varphi)$  with its imaginary unit  $i = \sqrt{-1}$  from the solutions of the wave equation, because  $\hat{\Phi}_m^*(\varphi) = e^{-im\varphi}$  and  $\hat{\Phi}_m(\varphi)\hat{\Phi}_m^*(\varphi) = 1$ . This step, consisting, thus, in elimination of inconvenient functions  $\hat{\Phi}_m(\varphi)$  from consideration, and resulting in getting rid of the independent variable  $\varphi$ , caused a series of the inevitable contradictions and of principle problems for QM.

The main result of all this "solution" is that Born's idea, becoming a principal postulate of QM, made it absolutely impossible to imagine and conceive the real structure of individual atoms, which are three-dimensional structures. The removing of the third coordinate  $\varphi$  from the trio of the spherical polar coordinates  $(r, \theta, \varphi)$  is unjustified step invalid for any reason [4].

Ignorance of the physical meaning of imaginary unit  $i$  and, hence, imaginary numbers in complex wave functions, led physicists to attributing to the polar-azimuthal functions a fictitious, erroneous, physical meaning. Evidently, they were so confident in the imaginary nature (unreality) of "imaginary" components of complex numbers, regarding this as a dogma, that none of them even began to doubt, that maybe these numbers are not imaginary, but are real? We think that for this reason physics are not even looked into the reference books on mathematics, in which they would have found that the *complex polar-azimuth functions determine the coordinates of the nodes and antinodes of the standing waves in the three-dimensional field-space*. This means that both components of the complex function are *real*. As a result, physicists-theorists unreasonably (arbitrary) attributed to polar-azimuth functions of the solutions of the wave equation the meaning of *electron atomic orbitals*.

Then, realizing that after all they cannot do without "imaginary" components of polar-azimuth functions, physicists, making combinations from the “real” and “imaginary” constituents, introduced the operation called *hybridization* of atomic orbitals. From that time the mixing of “real” and “imaginary” constituents of complex wave functions became a routine operation in quantum mechanics (QM) and quantum chemistry (QC).

Comprehensive analysis, which revealed unfoundedness of the concept of hybridization, is presented in [1]. Here we note only the principal points. The analysis has shown that hybridization is a pure *mathematical mixing* of qualitatively *opposite physical properties* (parameters) which are *physically immiscible* and, therefore, such a mixing is not observed in nature.

Let us put the question. Is it possible to mix physically together such two *qualitatively different entities* as electric and magnetic fields and to obtain some really existing third resulting field? Of course, such a physical blending is impossible in principle. In the case of hybridization of atomic orbitals, we faced the mixing of angular coordinates of *nodes* and *antinodes* of standing waves in a three-dimensional field-space that is nonsense. A hybrid of a

node and an antinode is an absurdity. It is merely a *mathematical trick* used by creators of QM at the earliest stage of its creation [1, 5].

Absolute lack of understanding of the real physical meaning of complex wave  $\Psi$ -functions with their “*imaginary*” components led in result, as we can see, to the construction of an abstract-mathematical theory – quantum mechanics. For this reason, within this theory, the true structure of atoms cannot be disclosed in principle. It is stated about the inability at the micro-level, which as believe is described by quantum theory, any visual representations, images, as in classical physics. Hence, there are no clear physical image of individual atoms, their internal structure, *i.e.*, the geometry of the disposition of nucleons in atoms; and the motion of electrons in the vicinity of the nuclei, at all, “shrouded in mist” because of the lack of the concept of trajectory, *etc.*

In fact, as shown by the experience of physics and follows from our analysis, as set out in books and articles, starting from 1995, “*real*” and “*imaginary*” parts of complex *wave* functions are both real. They represent two *qualitatively polar opposite* entities, in particular, *potential* and *kinetic* features of the *wave* process described by the wave function  $\hat{\Psi}(r, \theta, \varphi, t)$ :

$$\hat{\Psi} = \hat{\psi}(r, \theta, \varphi)\hat{T}(t) = \hat{R}_l(kr)\hat{Y}_{l,m}(\theta, \varphi)\hat{T}(\omega t) = (\psi_p + i\psi_k)\hat{T}(\omega t). \quad (3)$$

The imaginary unit  $i$  in complex wave functions, as, in particular, it is shown in the wave function (3), indicates this *qualitative contrast*.

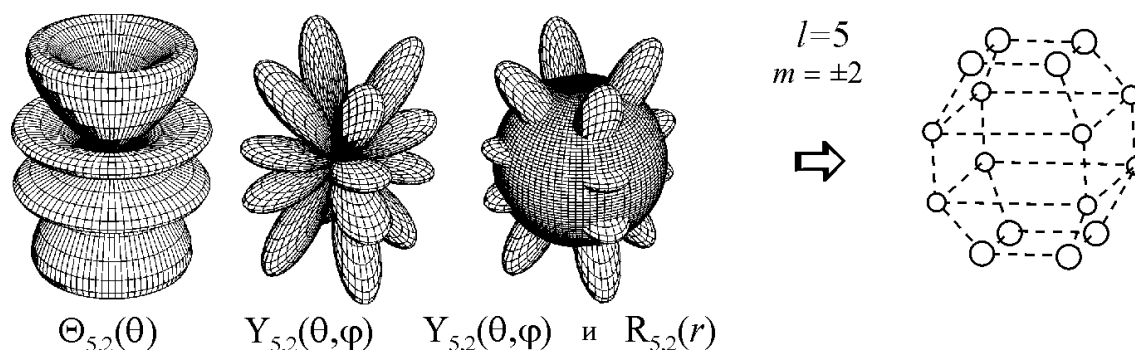
Polar-azimuthal factors  $\hat{Y}_{l,m}(\theta, \varphi)$  of the wave function define polar-azimuthal coordinates of *nodes and antinodes of standing spherical waves* located, respectively, on the potential and kinetic radial shells  $\hat{R}_l(kr)$ . *Antinodes* are the real entities just like nodes, being qualitatively their *polar opposites*.

Polar components  $\Theta_{l,m}(\theta)$  of the  $\hat{\Psi}$ -function define characteristic parallels of the disposition of nodes and antinodes (principal and collateral) on radial spheres (wave shells). Azimuthal components  $\hat{\Phi}_m(\varphi)$  define characteristic meridians of the disposition of nodes and antinodes. Together, potential and kinetic constituents of the polar-azimuthal functions  $\hat{Y}_{l,m}(\theta, \varphi)$  point at the spatial coordinates of zeros and extremes (nodes and antinodes) on the radial wave shells.

Polar-azimuth functions and their solutions are the same for both the standard wave equation and the Schrödinger equation. Radial solutions are different. Although, in the case of radial functions of the Schrödinger equation, we cannot talk at all about radial “solutions” because by and large they are not as such in the truest sense of the word. About this it is convincingly shown in [6]. However, in both cases, radial solutions determine the radii of potential and kinetic spherical shells, on which are located, respectively, the nodes and antinodes of the standing waves.

As an example, in Fig. 2, we present graphically a particular solution of the wave equation, which gives information about angular spatial coordinates of the disposition of nodes of standing waves, corresponding to quantum numbers  $l = 5, m = \pm 2$ . Qualitatively the same picture, but turned around the axis of symmetry Z by a certain angle, corresponds to angular spatial coordinates of antinodes (not shown), which, with a certain radial shift, are located between nodes.

Strict radial solutions are not presented here. The spherical surface (an image of the radial wave shell) was drawn in Fig. 2 arbitrarily to make it easier to specify the location of nodes in space on the wave shell (picture right). A complete table of nodes and other graphical representations of the solutions of the wave equation are shown in [7, 8].



**Fig. 2.** Graphical representation of one of the particular solutions of the wave equation.

Binary wave functions, reflecting the *symmetry-asymmetry* of polar opposite (*potential* and *kinetic*) properties of spaces yield information about the atomic structure and morphology (*symmetry*) of crystals [6]. Wherein, half-integer solutions, having in an equatorial plane the *any-fold symmetry*, reveal the true origin of the symmetries (five-, seven-, eight-fold, *etc.*) regarded in modern science as “*strictly forbidden by the mathematical laws of crystallography*” [9]. The superposition of elementary solutions of the wave equation determines the molecular structures.

Numerous data that we have obtained indicate that the well-known *constancy of angles between edges* (and facets) for all crystals of the same substance is conditioned by the wave nature of constituents – atoms. This conclusion is based on the fact that, as was found, the characteristic angles of crystals exactly coincide with the characteristic angles of wave polar-azimuthal functions [6]. Thus, at the macrolevel, in crystals, the spatial positions of crystal nodes are realized by the same pattern as at the microlevel, in individual atoms (in spatial positions of their nucleon nodes).

A comprehensive analysis also revealed a fundamentally important fact that chemical bonds (meaning their *direction*) in ordered structures, molecules, are realized along the strongly bound

nucleons of nearest neighboring nodes belonging to two wave shells (the last, external, and penultimate, internal) of each of the pairing atoms, but not caused by their electrons ("electron configuration of atoms"), as is commonly believed in modern physics like a matter of course. Thus, electrons determine only the *strength of chemical bonds, but not their directions*.

It is natural that on the fundamentally new basis, which we accepted in our research [6], the nature of Mendeleev's Periodic Law is also revealed from a new side. Namely, it was found that the periodicity of chemical properties of the elements is due to the likeness of the geometry of the spatial location of the nodes on the outer wave shells of the corresponding atoms [7, 10].

Conjugate potential-kinetic parameters, obeying every to one of the two algebras of signs [11], give the complete description of physical fields. The wave function  $\hat{\Psi}$ ,  $\hat{\Psi} = \Psi_p + i\Psi_k$ , reproduces mathematically the real image and binary character of the phenomenon. The letter  $i$  (imaginary unit) in the equation denotes the unit of negation, *i.e.*, points out the qualitatively opposite property  $\Psi_k$  (*kinetic*) with respect to  $\Psi_p$  (*potential*).

An existence of qualitatively opposite properties is the fundamental law of Nature [11], and the binary field of real conjugate parameters (numbers), relevant to these properties, takes account of this [12].

Two polar opposite in sign algebras, to which two qualitatively opposite components of the wave function obey, are the mathematical reflection of the dialectical law of affirmation-negation for qualitatively opposite binary judgments about the nature of any object or process. Here are examples of realization of two algebras of signs:

$$(\pm 1)(\pm 1) = +1, \quad (\pm 1)(\mp 1) = -1. \quad (4)$$

$$(\pm i)(\pm i) = -1, \quad (\pm i)(\mp i) = +1. \quad (5)$$

Two charges identical (in sign) repeal (the sign "+" in the left equality of (4) expresses this fact) and the charges opposite in sign attract (the sign "-" in the right equality of (4) reflects it). Such is an objective algebra of *central, longitudinal* fields of interaction.

In opposition to charges, two currents of the same sign (*i.e.*, the same direction)  $\pm i$  and  $\pm i$  attract through their magnetic (*transversal*) fields. The attraction (just like the repulsion) has the central character, so that the negative unit of the longitudinal field  $-1$  reflects it. Currents of opposite signs,  $\pm i$  and  $\mp i$ , repeal that is expressed by the measure  $+1$  (5).

Thus, '*imaginary*' numbers are not actually imaginary. All conjugate ("*real*" and "*imaginary*") numbers are *real*. In particular, the wave function, called in modern physics as a "complex" function comprising *real* and *imaginary* terms, in actual fact, is contained *only real components*, reflecting thus the potential-kinetic essence of rest-motion.

So, summing up, we emphasize once again that

the "*imaginary*" unit  $i$  is a symbol for a qualitatively polar opposite essence (property, number, parameter) obeying to the polar opposite ("*negative*") algebra of signs relative to the conventional ("*positive*") algebra.

Note also that to the disclosure of the true nature of complex numbers, we came by developing a mathematical representation of dialectical logic [11-13]. The resulting binary numerical field of dialectics, corresponding to this logic, does not differ by *the form from the representation* of complex numbers, but essentially differs, as was shown, *in content, in essence*.

Taking into account all of the above, we believe that the word "*imaginary*" as not reflecting reality will eventually be removed from mathematics and physics, harmonizing the mathematical structures with the laws of the Universe, and thus expanding the horizons of knowledge.

In the papers, which are given in References, the physical meaning of imaginary numbers and the role that "*imaginary*" unit  $i$  plays in them are considered from different perspectives and with different degrees of detail.

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*George Shpenkov*  
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