The First Precise Derivation of the Proton's Magnetic Moment

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The first precise derivation of the proton's magnetic moment is presented in this paper. Longitudinal and transversal exchange interactions and corresponding to them longitudinal and transversal exchange charges, originated from the Dynamic Model of Elementary Particles, were taken into account at the derivation.
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The current experimental value of the proton's magnetic moment, according to the 2006 CODATA internationally recommended values, is

$$\mu_p = 1.410606662(37) \cdot 10^{-26}\ J \cdot T^{-1}. \quad (1)$$

This value is approximately in 1.46 times modulo more than that one for the neutron. The reason of such a distinction between two observed quantities is not yet clearly understood by modern physics [1-18]. Resting on the Standard Model (SM), physicists do not know till now the *true nature of charge* responsible for magnetic properties of substance. The primordial problem of physics, the *problem on the mass and charge nature,* is not solving by the SM as based on *formal logic*.

A new approach based on *dialectical logic*, set forward in the last decade, has solved the above problem. The nature of mass and charge was uncovered in the framework of the Dynamic Model of Elementary Particles (DM) [19], which is a part of the Dialectical Model of the Universe. According to the DM, mass of elementary particles has the *associated* character and is the *measure of exchange* of matter-space-time, and the rest mass does not exist. The notion of *exchange* instead of interaction is one of the principal notions in the DM. The latter distinguishes the *longitudinal* exchange and the *transversal* exchange as two sides of the process of interaction of particles with surrounding fields and particles themselves. The *longitudinal exchange* is characteristic for *spherical fields* of particles at rest and motion. The *transversal exchange* is characteristic for *cylindrical fields* of moving particles only. The longitudinal (central) exchange is considered in detail in [19].

The rate of mass exchange defines the *exchange charge*, its dimensionality is $g \cdot s^{-1}$. Two notions of exchange charges correspond to two aforementioned types of exchange: the *longitudinal* ("*electric*") exchange charge and the *transversal* ("*magnetic*") exchange charge. The transversal charge is generated during the motion of particles.

A neutron, regarded in the DM as a proton-electron system, unstable in a free state, is an electrically neutral formation as a whole in which positive (longitudinal) exchange charge of the core of the neutron is compensated by the opposite, negative, transversal exchange charge of the electron being in a state of motion in the system. Accordingly, a moving neutron, as a neutral particle, does not generate the transversal exchange charge because transversal exchange with environment is absent. But, as in the case of the hydrogen atom, that is the proton-electron system as well, the constituent electron defines a negative magnetic moment of the neutron.

A naked proton has longitudinal exchange charge, equal in value to the minimal quantum of the rate of mass exchange, which is not compensated (as against of the neutron). Therefore, during motion it generates the transversal charge, which defines the transversal, magnetic field. Thus both exchanges and corresponding to them exchange charges, longitudinal and transversal, define the proton's magnetic moment that is shown here.

The spectrum of amplitude magnetic moments of nucleons (protons and neutrons) as dynamic (pulsing) microobjects, in accordance with the DM [19], is described by the formula

$$\mu = \frac{\upsilon_0}{c} q \frac{A\hat{e}_l(z_{p,s})}{z_{p,s}}, \tag{2}$$

where

$$A = r_0\sqrt{\frac{2hR}{m_0 c}} \tag{3}$$

is the constant,

$$\hat{e}_l(kr) = \sqrt{\pi kr/2}\,(J_{l+1/2}(k_e r) \pm iY_{l+1/2}(k_e r)), \tag{4}$$

$$k_e = \omega_e / c = 1/\lambdabar_e, \qquad z_{p,s} = k_e r \tag{5}$$

Here, υ_0 and r_0 are the Bohr speed and radius, respectively; c is the speed of light; q is the charge of *exchange* of the nucleon with environment, $g \cdot s^{-1}$ [19]; A is the constant; $J(kr)$ and $Y(kr)$ are Bessel functions; k_e is the wave number; ω_e is the oscillation frequency of the pulsating spherical shell of the proton equal to the fundamental "carrier" frequency of the subatomic and atomic levels; $z_{p,s}$ are roots of Bessel functions [23]. The subscript p in $z_{p,s}$ indicates the order of Bessel functions and s, the number of the root. The last defines the number of the radial shell. Zeros of Bessel functions define the radial shells with zero values of radial displacements (oscillations), *i.e.*, the shells of stationary states.

An *elementary quantum of the magnetic moment of a nucleon* in a node of the spherical field is equal to [24]

$$\mu = \frac{\upsilon_0}{c} q A_m, \tag{6}$$

where A_m is amplitude with which a nucleon as a whole oscillates in a node of the spherical wave field of exchange,

$$A_m = \lambdabar_e \sqrt{\frac{2Rh}{m_0 c}} = 2.73065189 \cdot 10^{-12}\ cm, \tag{7}$$

where

$$\lambdabar_e = c/\omega_e = 1.603886514 \cdot 10^{-8}\ cm \tag{8}$$

is the wave number, ω_e is the fundamental frequency of the subatomic level,

$$\omega_e = 1.869162534 \cdot 10^{18}\ s^{-1}. \tag{9}$$

The amplitude A_m is the characteristic amplitude of oscillations on the sphere of the wave radius ($z_{p,s} = kr = 1$). The Rydberg constant R is

$$R = \frac{R_\infty}{1 + m_e / m_0} = 109677.5833\ cm^{-1} . \qquad (10)$$

Other physical quantities used here are: the proton mass $m_0 = 1.672621637(83) \cdot 10^{-24}\ g$, the Planck constant $h = 6.62606896(33) \cdot 10^{-27}\ erg \cdot s$, and $c = 2.99792458 \cdot 10^{10}\ cm \cdot s^{-1}$.

The exchange charge *q*, being the measure of the *rate of exchange of matter-space-time*, or briefly the *power of mass exchange*, is defined as

$$q = \frac{dm}{dt} = S\upsilon\varepsilon , \qquad (11)$$

where *S* is the area of a closed wave surface separating the inner and outer spaces of an elementary particle, υ is the oscillatory speed of exchange at the separating surface, ε is the absolute-relative density. The exchange charge *q* and the Coulomb charge q_C (presented in the CGSE units) are related as

$$q = q_C \sqrt{4\pi\varepsilon_0} , \qquad (12)$$

where $\varepsilon_0 = 1\ g \cdot cm^{-3}$ is the absolute unit density.

The value (6) is the main constituent of the magnetic moment of nucleons (both a neutron and a proton). But in this formula, in the case of the moving naked proton, the total exchange charge *q* is equal to the minimal quantum of the rate of (longitudinal) mass exchange and an additional transversal exchange charge of the positive sign.

At the same time a nucleon, as a dynamic, pulsing microobject in accordance with the DM, oscillates with respect to its own center of mass with the amplitude (3). These perturbations in motion of a nucleon defined by the amplitude superimpose on the circular motion of the nucleon in the wave field of exchange. Hence, the second in value term to the nucleon's magnetic moment (6) is defined (in this case $z_{p,s} = z_{0,s}$) by the following formula

$$\delta\mu_1 = \frac{q\upsilon_0}{c} \frac{r_0}{z_{0,s}} \sqrt{\frac{2Rh}{m_0 c}} . \qquad (13)$$

The charge *q* in (13), for the case of a neutron, regarded as a proton-electron system, is defined by only electron's exchange charge, so that $q = -e$, which is mutually balanced with the opposite in sign central longitudinal proton's exchange charge.

For the naked proton, whose exchange with the surrounding field is not compensated with the opposite in sign exchange charge of the electron, because of the absence of the latter, the total exchange charge of the proton *q* is defined by the *longitudinal non-compensated positive exchange charge*, $+e = 1.702691582 \cdot 10^{-9}\ g \cdot s^{-1}$, and the supplementary *associated transversal exchange charge*, Δe_p, generated during its motion:

$$q = +e + \Delta e_p . \qquad (14)$$

Thus, the circular wave motion of the proton generates the magnetic (transversal) moment μ (6); and small deviations of the motion generate an additional magnetic moment (13). The theoretical value of the total magnetic moment of the proton $\mu_p(th)$ is defined thus by the following equation

$$\mu_p(th) = \frac{(e + \Delta e_p)\upsilon_0}{c} \left(\lambda\!\!\!^{-}_e + \frac{r_0}{z_{0,s}} \right) \sqrt{\frac{2Rh}{m_0 c}} \qquad (15)$$

In this formula, the unknown magnitude is the *supplementary associated charge* of the proton, Δe_p, generated during the non-compensated transversal exchange with environment of the moving proton. The roots of Bessel functions $z_{0,s}$, which define the discrete spectrum of wave shells of the proton, can be easy chosen from a series of roots obtained from solutions of the wave equation [25].

The formulas for the *associated transversal mass* m_τ and the *associated transversal charge* q_τ (at $\varepsilon_r = 1$) [24] are:

$$m_\tau = \frac{4\pi r_0^2 l\varepsilon_0}{1+4(k_r r_0)^2}, \tag{16}$$

$$q_\tau = \omega m_\tau = \frac{4\pi r_0 l\upsilon\varepsilon_0}{1+4(k_r r_0)^2}, \tag{17}$$

where l is a section of the cylindrical surface of the length l, $S = 2\pi r_0 l$, related to the cylindrical field around a trajectory of the moving proton.

Motion of a proton has the wave character and represents a cylindrical ray-wave. Therefore, we must take into account the supplementary associated mass and charge of the ray element l generated at the transversal exchange in the cylindrical field.

An element l of the ray can be defined by the following way. According to the approach developed in [24], we have to recognize that the *elementary quantum of the rate of mass exchange e* exists in the four states:

$$+e, \quad -e, \quad +ie, \quad -ie \tag{18}$$

The first two quanta have relation to the *longitudinal* ("electric") exchange, the rest two – to the *transversal* ("magnetic") exchange. An elementary transversal magnetic charge-flux at the level of the Bohr radius r_0 is

$$ei = \upsilon i\varepsilon_0 S = 2\pi r_0 li\upsilon\varepsilon_0, \tag{19}$$

where $\varepsilon_0 = 1\, g\cdot cm^{-3}$ is the absolute unit density, $e = 1.702691582\cdot 10^{-9}\ g\cdot s^{-1}$ is the charge of exchange of the proton with environment equal in magnitude to the electron exchange charge, and $r_0 = 0.52917720859\cdot 10^{-8}\ cm$ is the Bohr radius. Under the condition $\upsilon = c$, the element l of the ray-wave is minimal in value and, as follows from (19), is

$$l = \frac{e}{2\pi r_0 c\varepsilon_0} = 1.708182574\cdot 10^{-12}\ cm. \tag{20}$$

Hence, the *supplementary associated transversal* mass of the proton defined from (16) is

$$\Delta m_p = \frac{4\pi r_0^2 l\varepsilon_0}{1+4k_e^2 r_0^2} = 4.187602162\cdot 10^{-28} g, \tag{21}$$

The exchange charge in the DM [19, 24], $q = dm/dt$ (11), is regarded as the rate of mass exchange; its amplitude value is equal to the product of the associated mass by the fundamental frequency of exchange at the subatomic level ω_e,

$$q = m\omega_e. \tag{22}$$

Hence, the supplementary associated charge Δe_p, corresponding to the mass (21), is equal to

$$\Delta e_p = \Delta m_p \omega_e = 7.827309069 \cdot 10^{-10} \ g \cdot s^{-1}. \tag{23}$$

Thus, the total charge of exchange of the proton wave shell with environment is

$$q = e + \Delta e_p = 2.485422489 \cdot 10^{-9} \ g \cdot s^{-1}. \tag{24}$$

Let us turn now to the formula (15) and choose the root of Bessel functions $z_{0,s}$ entered in the second term of this expression. Similar as in the case of the derivation of the neutron's magnetic moment [21], we select the radial solution near the twelfth wave shell. Owing to the more uncertainty, we take the average value of the two nearest roots $z_{0,s}$, namely $a'_{0,11} = 32.95638904$, equal to the extremum of the eleventh potential spherical shell, and $y_{0,12} = 35.34645231$, which is equal to the zero of the twelfth kinetic shell. Under all above conditions, the formula (15) for the proton's magnetic moment takes the form

$$\mu_p(th) = \frac{(e + \Delta e_p)\upsilon_0}{c}\left(\lambda\!\!\!^{-}_e + r_0 \frac{(a'_{0,11} + y_{0,12})}{2(a'_{0,11} y_{0,12})}\right)\sqrt{\frac{2Rh}{m_0 c}}, \tag{25}$$

where $\upsilon_0 = \alpha c = 2.187691254 \cdot 10^8 \ cm \cdot s^{-1}$ (α is the fine-structure constant [28]).

The substitution of all numerical values for quantities entered in (25) yields the following theoretical values for the total magnetic moment of the proton and two its constituents:

$$\begin{aligned} \mu_p(th) &= (4.952571882 + 0.04790508144) \cdot 10^{-23} \ g \cdot cm \cdot s^{-1} = \\ &= 5.000476963 \cdot 10^{-23} \ g \cdot cm \cdot s^{-1} \end{aligned} \tag{26}$$

In the SI units, since $1T = 10^4 / \sqrt{4\pi} \ cm \cdot s^{-1}$, (26) is rewritten as

$$\mu_p(th) = (1.397094734 + 0.0135137738) \cdot 10^{-26} \ J \cdot T^{-1} = 1.410608508 \cdot 10^{-26} \ J \cdot T^{-1} \tag{27}$$

Thus, we have obtained the theoretical value μ_p, which practically coincides with the current "2006 CODATA recommended value" (1) accepted for the proton's magnetic moment. The absolute coincidence of the obtained theoretical value (27) with the averaged experimental (recommended) value (1) is easily achieved if one introduces a small empirical coefficient $1/\beta$ for the second term. Such an adjustment is justified in the framework of the approach accepted here, because it corrects indeterminacy in the weight contribution each of two selected shells (roots of Bessel functions). The coefficient $1/\beta$ takes into account this circumstance.

Thus finally, the formula for the magnetic moment of the proton (25) takes the form

$$\mu_p(th) = \frac{(e + \Delta e_p)\upsilon_0}{c}\left(\lambda\!\!\!^{-}_e + r_0 \frac{1}{\beta}\frac{(a'_{0,11} + y_{0,12})}{2(a'_{0,11} y_{0,12})}\right)\sqrt{\frac{2Rh}{m_0 c}} \tag{28}$$

At $\beta = 1.000136546$, we arrive at

$$\mu_p(th) = (1.397094734 + 0.013511928) \cdot 10^{-26} \ J \cdot T^{-1} = 1.410606662 \cdot 10^{-26} \ J \cdot T^{-1}, \tag{29}$$

i.e., at the complete coincidence of the experimental value of the magnetic moment of the proton (1) with theoretical (29).

Thus for the first time a precise derivation of the proton's magnetic moment is realized with due account of wave behaviour of the particle. Along with the previous derivation of electron's and neutron's magnetic moments [20, 21], and the Lamb "shifts" in the hydrogen atom [22], this work is the next stringent test for the new concepts, originated from the DM, on the associated nature of mass of elementary particles and the exchange nature of charges turned out to be the rate of mass exchange of the dimensionality $g \cdot s^{-1}$.

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