Important Results of
Analyzing Foundations of Quantum Mechanics

Leonid G. Kreidik

Polytechnic Academy, Minsk, BELARUS

and George P. Shpenkov

Institute of Mathematics & Physics, University of Technology and Agriculture,

Al. Kaliskiego 7, Bydgoszcz 85-796, POLAND; e-mail shpenkov@janmax.com

The foundations of quantum mechanics are analyzed, and some problems that appeared during its creation and remain unsolved today are emphasized. It is shown that the introduction of variable wave number *k*, depending on electron coordinates, and the omission of the azimuth part of the wave function $\hat{\psi}$, were erroneous. Including the azimuth factor of the wave function and taking into consideration the constant value of *k* result in wave-equation solutions showing a discrete nodal structure of intratomic space, and arrives at a periodic-nonperiodic law for behavior of atomic structures. Viewing atoms as quasi-spherical neutron molecules makes it possible to understand characteristic features of the periodic table.

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1. Introduction

In the first decades of the 20th century, classical mechanics met with problems in the description of motion at the microlevel, creating the necessity to develop equations of motion on the basis of wave concepts. The first step was made by Schrödinger, who introduced the wave function $\hat{\Phi}$, which is in general complex, the complexity being denoted here by the sign ^ above the symbol. At that time, the meaning of the wave function was not clearly understood. It was represented in the form of a product $\hat{\Phi} = \hat{\psi}(x, y, z)e^{i\omega t}$ of a spatial function $\hat{\psi}(x, y, z)$ and a time factor $e^{i\omega t}$, where $\omega = W / \hbar$, with *W* being energy

On the basis of optical analogies [1], Schrödinger built the wave equation for the spatial function:

$$\frac{\partial^2 \hat{\psi}}{\partial x^2} + \frac{\partial^2 \hat{\psi}}{\partial y^2} + \frac{\partial^2 \hat{\psi}}{\partial z^2} + \frac{2mE}{\hbar^2} \hat{\psi} = 0$$

or

$$\Delta \hat{\psi} + k^2 \hat{\psi} = 0 \tag{1.1}$$

where

$$k = \pm\sqrt{2mE / \hbar^2} \tag{1.2}$$

is the wave number; *E* is the kinetic energy of the electron, presented as the difference between total energy *W* and potential energy *U* depending on *x, y, z* coordinates. Schrödinger assumed (and this assumption is generally accepted now), that the wave motion of the electron was around a nucleus with charge Ze. In terms of radius $r = \sqrt{x^2 + y^2 + z^2}$, the potential is

$$U(x, y, z) = -Ze^2 / 4\pi\varepsilon_0 r \tag{1.3}$$

The condition (1.3) imposed on Schrödinger's equation a definite type of solution with the wave number *k* dependent on coordinates. But all experience in physics points to the fact that wave number remains a constant or varies only insignificantly in space, both over large volumes of cosmic space, and at the subatomic level. A field potential does not influence the wave speed in any practical way. So Schrödinger's variable wave number should be questioned, because the potential function cannot influence the wave speed or, consequently, the wave number.

Apparently, Schrödinger was unable to identify correct boundary conditions to specify the otherwise indefinite wave function. When he first began to study these questions, he noticed that a simplification to 'no' boundary conditions seemed necessary [2]. But not having been sufficiently schooled in mathematics, he could not understand how fundamental oscillations could occur without boundary conditions. He later wrote that a more complicated form of coefficients [containing $U(x, y, z)$] must provide information that is usually given by boundary conditions. Unfortunately, he was mistaken in this, because the potential function actually destroys the wave equation.

2. Conceptual Flaws in Quantum Mechanics

Indeed, the wave function $\hat{\psi}(x, y, z)$ is presented in the form of the product of radial $R_{n,l}(r)$, polar $\Theta_{l,m}(\theta)$, and azimuth $\hat{\Phi}_m(\varphi)$ factors:

$$\hat{\psi} = R_{n,l}(r)\Theta_{l,m}(\theta)\hat{\Phi}_m(\varphi), \text{ or } \hat{\psi} = R_{n,l}(r)\hat{Y}_{l,m}(\theta, \varphi) \tag{2.1}$$

where

$$\hat{Y}_{l,m}(\theta, \varphi) = \Theta_{l,m}(\theta)\hat{\Phi}_m(\varphi) \tag{2.2}$$

denotes the polar-azimuth function.

For the radial factor the potential function *U* from (1.3) implies the differential equation

$$\frac{d^2 R_{n,l}(r)}{dr^2} + \frac{2}{r}\frac{dR_{n,l}(r)}{dr} + \left[\frac{2m}{\hbar^2}\left(W + \frac{Ze^2}{4\pi\varepsilon_0 r}\right) - \frac{l(l+1)}{r^2}\right]R_{n,l}(r) = 0 \tag{2.3}$$

The solutions of (2.3) for $R_{n,l}(r)$ is a functional series which in general diverges*; i.e.* $R_{n,l}(r) \rightarrow \infty$ for some *r*. Addressing this problem, Schrödinger together with H. Weyl (German mathematician, 1885-1955) found the condition under which the series terminates with a finite number of terms and finite values of

$R_{n,l}(r)$ for all *r*. The condition is that the electron total energy *W* has a numerical value expressible with infinite mathematical accuracy in the form

$$W = Z^2 e^2 / 8\pi\varepsilon_0 a n^2 \qquad (2.4)$$

where *a* is the Bohr radius and *n* is the principal quantum number. It follows that any tiny variation of electron energy, for example at the level $\Delta W = 10^{-137} W$, results again in $R_{n,l}(r) \to \infty$. Thus, the condition (2.4) is a mathematical manipulation far removed from reality. Moreover, the uncertainty principle – the basis of quantum mechanics – excludes any such accuracy.

Further, the wave function (2.1) is complex. Misunderstanding of the nature of the imaginary part of complex numbers has generated definite difficulties. These have not been solved, but only bypassed, by formally eliminating the azimuth factor from the wave function. This move predetermined the introduction of the modulus squared of the wave function as the wave density of probability for distribution of electron mass and charge in the space surrounding the nucleus of the atom. Until now, it has been assumed that the electron charge is distributed throughout the intratomic space. But at the same time, it follows from the potential function (1.3) used in Eq. (1.1) that the electron charge is concentrated in a point. This is a logical conflict.

Note too that the modulus squared $|\hat{\psi}|^2 = \hat{\Psi}\hat{\Psi}^*$ is not itself a solution to the wave equation. Such a 'small item' did not attract attention at that time. Instead, $|\hat{\Psi}|^2$ was interpreted as fundamental. This allowed the 'phase' aspect of the wave function to be ignored, and it allowed $|\hat{\Psi}|^2$ to be interpreted as a probability density, the integral $\int |\psi|^2 \, dx \, dy \, dz = 1$ determining the unit probability that the electron is located in the intratomic space. As a result, only the appearance of a solution to the boundary conditions problem, but not the solution itself, was generated.

Understanding the conditionality of such an interpretation, Schrödinger noted in 1952 [2] that "We must agree that our concept of material reality is more fuzzy and indefinite than it was many years ago. We know plenty of the interesting things and every day we learn new details. However, we are unable to construct a clear picture, easily imagined, with which all physicists could agree. Physics is experiencing a deep ideological crisis." Only the basic sense of his expressions has been appreciated until now. The subsequent development of quantum mechanics eloquently points to the fact that the interpretation of the wave function was a problem for physicists, and it still remains so, although many researchers understand its conditional character.

It is assumed that the modulus squared of the wave function determines an 'electron density', closely related to the electron potential function (1.3). However, a formal introduction of the potential function in Schrödinger's equation does not quite mean that the polar-azimuth distribution (2.2) has a relation to electrons. As a pure mathematical function, (2.2) is not related to the electron potential function (1.3) (or to any other one either), and $\hat{Y}_{l,m}(\theta, \varphi)$ is independent of boundary conditions.

In the first publications, the functions $\hat{Y}_{l,m}(\theta, \varphi) = \Theta_{l,m}(\theta)\hat{\Phi}_m(\varphi)$ (see Table 2.1) were presented as graphs mainly

as cross-sections of their modulus squared in the planes passing through *Z*-, *Y*-, and *X*-axes {see, for example, Fig. 2.2.17 in [3]}. Actually, their modulus graphs are the surfaces of rotation of these functions about the polar *Z*-axis, as it is presented here in Table 2.2. As a result, attention was focused on the distribution of so-called 'electron density' (electron 'orbitals') around different axes lying in the planes of these cross-sections, all of which, without exception, were presented in a cigar-shaped form. However, the later is correct only at $m = 0$. In the other cases ($m \neq 0$), such representation leads to gross errors.

Table 2.1. Reduced polar-azimuth functions $\tilde{Y}_{l,m}(\theta, \varphi)$

l	*m*	$\tilde{Y}_{l,m}(\theta, \varphi) = \tilde{\Theta}_{l,m}(\theta)\hat{\Phi}_m(\varphi)$
0	0	1
1	0	$\cos\theta$
	± 1	$\sin\theta \exp(\pm i\varphi)$
2	0	$\cos^2\theta - 1/3$
	± 1	$\sin\theta\cos\theta \exp(\pm i\varphi)$
	± 2	$\sin^2\theta \exp(\pm 2i\varphi)$
3	0	$\cos\theta(\cos^2\theta - 3/5)$
	± 1	$\sin\theta(\cos^2\theta - 1/5)\exp(\pm i\varphi)$
	± 2	$\sin^2\theta\cos\theta \exp(\pm 2i\varphi)$
	± 3	$\sin^3\theta \exp(\pm 3i\varphi)$
4	0	$\cos^4\theta - 6/7\cos^2\theta + 3/35$
	± 1	$\sin\theta\cos\theta(\cos^2\theta - 3/7)\exp(\pm i\varphi)$
	± 2	$\sin^2\theta(\cos^2\theta - 1/7)\exp(\pm 2i\varphi)$
	± 3	$\sin^3\theta\cos\theta \exp(\pm 3i\varphi)$
	± 4	$\sin^4\theta \exp(\pm 4i\varphi)$
5	0	$\cos\theta(\cos^4\theta - 10/9\cos^2\theta + 5/21)$
	± 1	$\sin\theta(\cos^4\theta - 2/3\cos^2\theta + 1/21)\exp(\pm i\varphi)$
	± 2	$\sin^2\theta\cos\theta(\cos^2\theta - 1/3)\exp(\pm 2i\varphi)$
	± 3	$\sin^3\theta(\cos^2\theta - 1/9)\exp(\pm 3i\varphi)$
	± 4	$\sin^4\theta\cos\theta \exp(\pm 4i\varphi)$
	± 5	$\sin^5\theta \exp(\pm 5i\varphi)$?)
6	0	$\cos^6\theta - 15/11\cos^4\theta + 5/11\cos^2\theta - 5/231$
	± 1	$\sin\theta\cos\theta(\cos^4\theta - 10/11\cos^2\theta + 5/33)\exp(\pm i\varphi)$
	± 2	$\sin^2\theta(\cos^4\theta - 6/11\cos^2\theta + 1/33)\exp(\pm 2i\varphi)$
	± 3	$\sin^3\theta\cos\theta(\cos^2\theta - 3/11)\exp(\pm 3i\varphi)$
	± 4	$\sin^4\theta(\cos^2\theta - 1/11)\exp(\pm 4i\varphi)$
	± 5	$\sin^5\theta\cos\theta \exp(\pm 5i\varphi)$
	± 6	$\sin^6\theta \exp(\pm 6i\varphi)$

Sometimes, the modulus squared of real components of the complex function is implicitly used. However, by tradition, the corresponding graphs in the equatorial plane are presented in the cigar-shaped form, not always correctly.

Polar-azimuth graphs represent the surfaces of rotation around only polar *Z*-axis; hence, they cannot be treated as the volumetric objects in the 'electron density' form. An imaginary rotation of the cross-sections of polar diagrams around their axes of symmetry (lying in planes of these cross-sections) states a few

polar axes of an atom that is nonsense: a physical system, as whole, can have only a general polar axis of rotation.

Let us consider this question in detail. Polar-azimuth functions are presented in Table 2.1 (for simplicity, coefficients of functions to the higher powers have been taken to be equal to unity). Since the distribution of the modulus of the wave function is the qualitatively the same as the distribution of its modulus squared, we will consider only the distribution of modulus. On the basis of the data in Table 2.1, the corresponding graphs of polar-azimuth modulus $|\hat{Y}_{l,m}(\theta,\varphi)|$ are drawn in Table 2.2 (drawings of $|\hat{Y}_{l,m}(\theta,\varphi)|^2$ can be found, *e. g.*, in [4]).

Table 2.2. Polar levels of surfaces - diagrams of the modulus $|\hat{Y}_{l,m}(\theta,\varphi)|$.

	$m=0$	$m=\pm1$	$m=\pm2$	$m=\pm3$	$m=\pm4$	$m=\pm5$
s-level $l=0$						
p-level $l=1$						
d-level $l=2$						
f-level $l=3$						
g-level $l=4$						
h-level $l=5$						

Graphs $|\hat{Y}_{l,m}(\theta,\varphi)|$ for $m=0$ have the cigar-shaped form (with additional tori of collateral maxima, beginning from $l=2$), and in the other cases, when l and $m=1,2,...$ they represent toroidal surfaces of rotation of the cigar-shaped cross-sections. Thus, the modulus of a polar-azimuth function is characterized by main and collateral extremes. They indicate polar coordinates (angles θ) of extremal values of $\psi(x,y,z)$ *on the corresponding spheres* of radial factors of the function.

The simplest graph of $|\psi(x,y,z)|$ occurs at $l=0$ and $m=0$ (Fig. 2.1). Extremes of the $|\psi|$-function are on a sphere, which

we represent conditionally by the spherical layer-vicinity enveloping the spherical extremum.
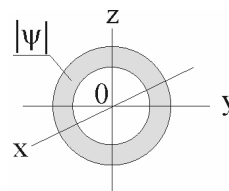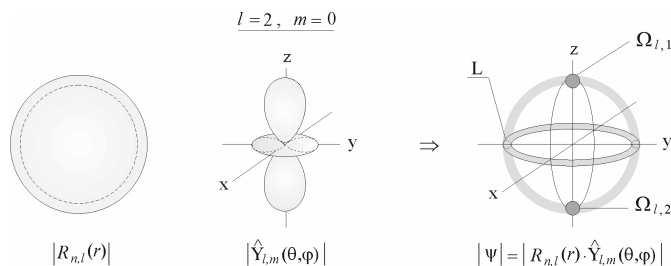

Figure 2.1. Distribution of domains of the maximum of the wave function modulus at $l=0$ and $m=0$ in the spherical space-field of an atom; O is the origin of coordinates; $|\psi|$ is the spherical layer vicinity of the maximum.

It follows from the *p*-distribution of the $|\psi|$-function, that for $l=1$ and $m=0$ two polar maxima $\Omega_{l,1}$, $\Omega_{l,2}$ are found at the spherical (radial) shell; they are presented in Fig. 2.2 by *spherical shells of a small incremental radius*.

$$|R_{n,l}(r)| \qquad |\hat{Y}_{l,m}(\theta,\varphi)| \qquad |\psi| = |R_{n,l}(r)\cdot\hat{Y}_{l,m}(\theta,\varphi)|$$
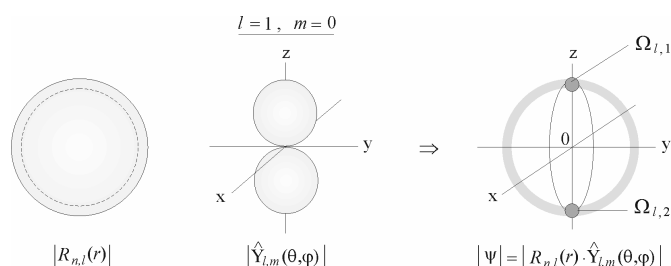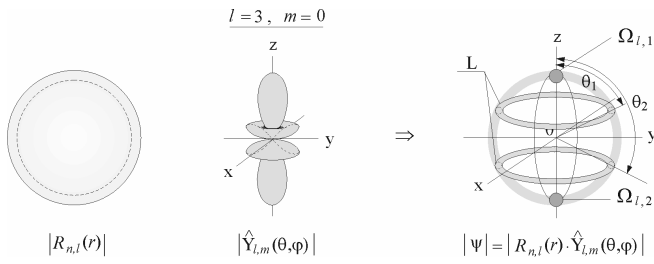
Figure 2.2. Distribution of domains of maxima of the wave function modulus $|\psi|$ at $l=1$ and $m=0$ in the spherical space-field of an atom. $|R_{n,l}(r)|$ is the spherical layer-vicinity of the maximum of the radial factor (an internal surface of the radial spherical layer is presented by a dash circumference); $|\hat{Y}_{l,m}(\theta,\varphi)|$ is the surface of the modulus of the polar-azimuth factor; $\Omega_{l,1}$, $\Omega_{l,2}$ are locations of polar maxima of $|\psi|$.

Such an image of the distribution cannot be interpreted as the distribution of *electron density* since, as was mentioned above, the polar-azimuth function (2.2) is independent of any physical content (parameter).

$$|R_{n,l}(r)| \qquad |\hat{Y}_{l,m}(\theta,\varphi)| \qquad |\psi| = |R_{n,l}(r)\cdot\hat{Y}_{l,m}(\theta,\varphi)|$$

Figure 2.3. Distribution of domains of maxima of the wave function modulus $|\psi|$ at $l=2$ and $m=0$ in the spherical space-field of an atom; $\Omega_{l,1}$, $\Omega_{l,2}$ are locations of the polar maxima of $|\psi|$; L is the annular equatorial location of the collateral maximum of the wave function modulus.

At $l=2$, $m=0$, *d*-distribution of $|\psi|$-function (Fig. 2.3) has at the radial shell two other polar maxima $\Omega_{l,1}$, $\Omega_{l,2}$ and a ring of the collateral maximum at the equator. This distribution cannot be referred to electron density either.

$l = 3 , \; m = 0$

$|R_{n,l}(r)|$ $|\hat{Y}_{l,m}(\theta,\varphi)|$ $|\psi| = |R_{n,l}(r) \cdot \hat{Y}_{l,m}(\theta,\varphi)|$
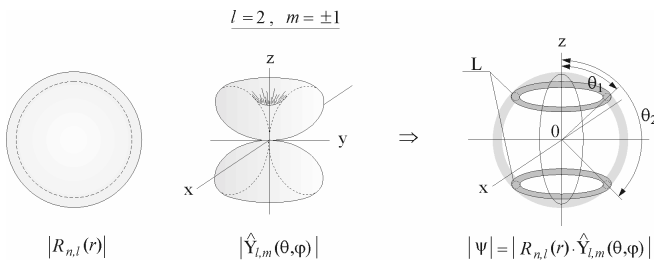
Figure 2.4. Distribution of domains of maxima of the wave function modulus $|\psi|$ at $l = 3$ and $m = 0$ in the spherical space-field of an atom; $\Omega_{l,1}, \Omega_{l,2}$ are locations of the polar maxima of $|\psi|$; L is the annular locus (of middle latitudes) of collateral maxima of the wave function modulus; $\theta_1 = 63^0 26'5''.82$, $\theta_2 = 116^0 33'54''.18$.

At $l = 3$ and $m = 0$ (Fig. 2.4), we have the next pair of polar maxima $\Omega_{l,1}, \Omega_{l,2}$ and two rings of collateral maxima of middle latitudes. Correspondingly, at $l = 4$ and $m = 0$, we arrive also at two polar maxima $\Omega_{l,1}, \Omega_{l,2}$, but at three rings of collateral maxima. One ring is at the equator and two other, defined by the angles $\theta_1 = 49^0 6'23''.78$ and $\theta_2 = 130^0 53'36''.22$, are in the middle latitudes.

Finally, at $l = 5$ and $m = 0$, we have one more pair of polar maxima $\Omega_{l,1}, \Omega_{l,2}$ and four rings of collateral maxima. The first pair of rings, nearest to the poles, is defined by the angles $\theta_1 = 40^0 5'17''.11$ and $\theta_4 = 139^0 54'42''.89$. The second pair is defined by the angles $\theta_2 = 73^0 25'38''.32$ and $\theta_3 = 106^0 34'21''.68$. And so forth.

At $l = 2$, $m = \pm 1$, the *d*-distribution of the $|\psi|$-function (Fig. 2.5) is characterized by two rings of main maxima (at the spherical shell, in the domain of middle latitudes). At $l = 2$, $m = \pm 2$, the *d*-distribution of the $|\psi|$-function (Fig. 2.6) is represented by one ring of the main maximum.

$l = 2 , \; m = \pm 1$

$|R_{n,l}(r)|$ $|\hat{Y}_{l,m}(\theta,\varphi)|$ $|\psi| = |R_{n,l}(r) \cdot \hat{Y}_{l,m}(\theta,\varphi)|$

Figure 2.5. Distribution of domains of maxima of the wave function modulus $|\psi|$ at $l = 2$ and $m = \pm 1$ in the spherical space-field of an atom; L is the annular locus (of middle latitudes) of main maxima of $|\psi|$; $\theta_1 = 45^0$ and $\theta_2 = 135^0$.

It is also possible to analyze other distributions. But in all cases at $m \neq 0$ (in accord with the strict interpretation of quantum mechanical solutions of Eq. (1.1) in terms of $|\psi|$ or $|\psi|^2$ that leads to the same result), we have *rings* of main and collateral maxima *at the radial spherical surfaces*. However, we do not have cigar-shaped volumes (cut out from the figures of rotation along the *z*-axis) that are commonly used in quantum mechanics.

$l = 2 , \; m = \pm 2$

$|R_{n,l}(r)|$ $|\hat{Y}_{l,m}(\theta,\varphi)|$ $|\psi| = |R_{n,l}(r) \cdot \hat{Y}_{l,m}(\theta,\varphi)|$
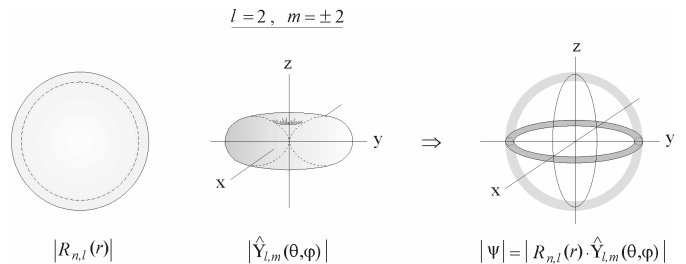
Figure 2.6. Distribution of domains of the maximum of the wave function modulus $|\psi|$ at $l = 2$ and $m = \pm 2$ in the spherical space-field of an atom; L is the equatorial ring of the main maximum of $|\psi|$.

We considered the distribution of the modulus of the wave function, selecting only extremes in a spherical space-field, where most probably the matter (particles constituent of an atom, *etc.*) must be localized. Because nobody knows what kind of particles are there, we will call them *X*-particles.

The image of distribution of *X*-particles in a spherical wave space-field was given above in strict correspondence with solutions of Schrödinger's equation (1.1) and David Bohm's interpretation of the modulus squared of the wave function [5].

In full agreement with the ideology accepted by quantum mechanics, the modulus squared $|\hat{\psi}|^2$ (or the modulus $|\hat{\psi}|$ that leads to the same result) of the wave function has been taken into consideration. Hence the above-presented analysis is, generally speaking, the analysis of semisolutions of the basic equation of quantum mechanics (1.1), since quantum mechanics does not give solutions for $\hat{\psi}$. The important conclusions that must be emphasized are as follows:

1) Because the *s*-state (Fig. 2.1) is characterized by spherical symmetry, quantum mechanics attributes a similar symmetry to the hydrogen atom.

First, what relation do all considered solutions have, basically, to the hydrogen atom? From where does it follow?

Second, even if we will agree with such a supposition as to the relation of the *s*-state to *H*-atom, nevertheless, the spherical symmetry of the hydrogen atom is a myth. *The hydrogen atom, as a paired proton and electron, does not possess the spherical symmetry* that defines corresponding angular and magnetic moments. Moreover, the proton in turn has the discrete structure with the polar axis of symmetry [6] that manifests itself in the fine and superfine structure of nucleon spectra, *etc.*

2) Every radial shell of atoms with the wave number *m*=0 is represented by two polar maxima and rings of collateral maxima, which means that these shells are *characterized by the axis of the infinite-fold symmetry.* The main and collateral maxima-rings at $m \neq 0$ give the same infinite-fold symmetry. But, such symmetry cannot form the discrete atomic spaces. Hence, solutions obtained do not reflect the discrete feature of matter.

Thus, *quantum mechanics solutions*, in their modern form, *contradict reality* because, on the basis of these solutions, the existence of crystal substances-spaces is not possible.

3) It is a principle as well that the wave function itself is always characterized by three arguments: ρ, θ, and φ (in the spherical polar frame of reference), independently on its use in concrete cases to describe different wave processes. Therefore, it is im-

possible to agree with such a mathematical operation by which the *azimuth angle* φ *is 'cut off'* from the wave function, because the definite information, which implies the wave equation and the function itself, is rejected as a result.

4) Introduction of the potential function (1.3) in the wave equation, which results in dependence of the wave number k on the Coulomb potential, *generates divergences* that do not have a physical justification. They are eliminated in an artificial way.

5) In a theory of wave processes and oscillations, mutually conjugated parameters U and V (as for example, electric and magnetic vectors E and H) represent, usually, in the form of complex function $\hat{\Psi} = U + iV$. And a question about a meaning of $\hat{\Psi}$ does not arise because functions U and V have, in accord with their definitions, a definite physical meaning. Quantum mechanics does not separately consider the second ('imaginary') member in the complex wave function because the nature of 'imaginary' numbers is unknown for it.

6) In modern computer programming languages, there exists a command 'FORWARD' by which it is possible to offer a procedure without a concrete physical filling of it, anticipating events. In this sense, Schrödinger's equation (1.1) is a logical formation similar to the directive 'FORWARD', which accepts the real-science filling only after definition of the wave function on the basis of initial notions, independent of the wave equation. Unfortunately, as we see, the initial concepts have been incorrectly introduced by quantum mechanics, which resulted in the questionable atomic model commonly used now.

3. The Right Presentation of the Wave Equation and its Effects

As follows from the above described, the variable wave number k destroys the wave equation for $\hat{\psi}$. Moreover, elimination of the third azimuth coordinate φ in three-dimensional presentation of results, caused by operation only with $|\hat{\psi}|^2$, has led to absurdity.

To correct the faults of quantum mechanics found here, it is necessity to carry out the following:

1) The variable factor k (1.2) in Eq. (1.1) must be replaced with the constant wave number $k = \omega / c$, where ω is the **fundamental 'carrying' frequency** of the wave field at the corresponding level of space, *c* is the speed of light.

2) We must face the necessity to find $\hat{\psi}$, rather than $|\hat{\psi}|^2$ or $|\hat{\psi}|$ that does not contain the azimuth coordinate φ. Namely, the azimuth factor $\hat{\Phi}(\varphi)$ must be taken into consideration along with radial $R(r)$ and polar $\Theta(\theta)$ factors of the wave function (2.1).

Under the above-accepted conditions, the space factor $\hat{\psi}(x, y, z)$ of the general solution will satisfy

$$\Delta\hat{\psi} + \frac{\omega^2}{c^2}\,\hat{\psi} = 0 \qquad (3.1)$$

In this case, the differential equation for the radial factor $R(r)$ (considered in detail in [6]) is

$$\rho^2\,\frac{d^2 R_l}{d\rho^2} + 2\rho\,\frac{dR_l}{d\rho} + \left[\rho^2 - l(l+1)\right]R_l = 0 \qquad (3.2)$$

where $\rho = kr$. Thus, the wave functions in two wave equations, (1.1) and (3.1), are distinguished by various radial functions, described by different equations, (2.3) and (3.2) correspondingly, whereas polar-azimuth factors are equal, to within normalizing factors.

We do not consider here the nature of the ingredients ω and c of the k-parameter, because on the whole their constant values do not influence solutions of Eq. (3.1). The value of the fundamental frequency ω determines only the absolute scale of all parameters at the corresponding level of space. At the atomic and subatomic levels (see [6]), it is equal to

$$\omega_e = 1.86916197 \cdot 10^{18}\ \text{s}^{-1} \qquad (3.3)$$

The wave radius corresponding to (3.3) is

$$\lambda_e = c / \omega_e = 1.603886998 \cdot 10^{-8}\ \text{cm} \qquad (3.3a)$$

As we can see, λ_e is equal to one-half of mean value of the interatomic distance in crystals (in terms of the generally accepted atomic model); that is not a random coincidence.

Extremes and zeros of the polar factors $\Theta_{l,m}(\theta)$ determine (as shown in Figs. 2.2 – 2.6 on the right) *circumferences-parallels* at the radial spheres. A continuous set of such circumferences forms the polar surface of rotation (Table 2.2).

Azimuth factors $\hat{\Phi}_m(\varphi) = \exp(\pm im\,\varphi) = \cos m\,\varphi \pm i\sin m\,\varphi$ (the role of which was neglected by conventional quantum mechanics) determine *azimuth planes*, where the wave function $\hat{\psi}(x, y, z)$ takes extremal or zero values. Such planes pass through the polar Z-axis and form meridians at the sphere defined by the radial function. (The meaning of two components of complex functions (numbers) and the meaning of number i are revealed in the works [6, 8])

Thus, the *polar-azimuth function* $\hat{Y}_{l,m}(\theta, \varphi) = \Theta_{l,m}(\theta)\hat{\Phi}_m(\varphi)$ *selects at the characteristic radial spheres* of the radial function $R_l(r)$ the *point nodes-extremes and nodes-zeros* (kinetic and potential nodes). Potential ('real') factors $\hat{Y}_{l,m}(\theta, \varphi)_p$ (Table 3.1) of polar-azimuth functions $\hat{Y}_{l,m}(\theta, \varphi)$ are defined by the formula

$$\hat{Y}_{l,m}(\theta, \varphi)_p = \Theta_{l,m}(\theta)\cos m\,\varphi \qquad (3.4)$$

kinetic ('imaginary') components – by the formula

$$\hat{Y}_{l,m}(\theta, \varphi)_k = \Theta_{l,m}(\theta)\sin m\,\varphi \qquad (3.4a)$$

Extremes (positive or negative) are between zeros-nodes, exactly repeating zeros-nodes geometry. Therefore, it is sufficient to show only the geometry of zeros-nodes. This is drawn in Table 3.2 (here $n = l$). It is natural to assume that if extremes determine maxima of motion and, consequently, are 'kinetic' points of space, then zeros should be attributed to the 'potential' points of rest.

It is reasonable to assume that the distribution of nodes of the wave spherical space, described by the wave equation (3.1) and

presented in Table 3.2, also defines the distribution of particles of matter, if a material space is considered. As far as any particles, obviously, they have to be disposed in nodal points of the wave space. The more so, the wave equation of space [6]

$$\Delta \hat{\Pi} = \partial^2 \hat{\Pi} / \partial \tau^2 \qquad (3.5)$$

where $\hat{\Pi} = \hat{\psi}\hat{T}(\tau)$, and $\hat{\psi}$ is the space factor of the general solution, satisfying to the Eq. (3.1), does not describe the motion of isolated objects, but it describes the wave process at the definite level of space on the whole. Therefore, wave solutions of space allow us to determine space structures as unified systems, *i.e.* *wave equations of space are equations of microsystems*.

The question arises: what kinds of particles are localized in nodes of space at the atomic level – neutrons, protons, electrons, or all together? Analyzing the structure of crystals at the end of 18th century, R.J. Haüy (1743-1822) [7] came to the conclusion that it is necessary to consider atoms as elementary molecules, the internal structure of which determines the crystal structure of solids.

As masses of atoms are multiple of the neutron mass (or hydrogen atom mass), following Haüy's ideas makes it reasonable to suppose that any atom, like the elementary Haüy's molecule, is the neutron (*H*-atom) molecule.

Actually, it was shown by comprehensive analysis of direct and indirect consequences of solutions of the wave equation (3.1) [6], that nodes of intratomic space are completed by neutrons (*H*-atoms); therefore, atoms should be considered as **neutron (H-atom) quasispherical multiplicative molecules**. The word 'multiplicative' means that particles, constituted of these elementary molecules, must be coupled by strong bonds, which we call the multiplicative bonds. Then, it is reasonable to call ordinary molecules with relatively weak (so-called chemical) bonds composite or **additive molecules**; *e.g.* if deuterium *D* is the multiplicative molecule then the hydrogen molecule H_2 is the additive one. Thus, it is possible to assume that Table 3.2 shows the actual picture of distribution of nodes-extremes, corresponding to Haüy's elementary molecules.

Principal azimuth nodes of the wave space of atoms are marked by ordinal numbers. These numbers coincide with the ordinal numbers of elements of Mendeleev's periodic table. The quantity of neutrons, localized in one node, is equal to or less than two. Collateral nodes, designated in the Table 3.2 by smaller white circles, are partially vacant; these possibly provide conditions for the intratomic movement of neutrons or protons. For example, one of the isotopes of $_{14}Si$ has four spherical neutron shells, the principal nodes of which are completed (contain 28 neutrons), but two collateral nodes of the outer shell ($n = 3$, $m = \pm 1$, see Table 3.2 where $n = l$) are vacant. The latter, apparently, determine the semiconductor properties of $_{14}Si$.

Table 3.1. Reduced polar-azimuth potential functions $\tilde{Y}_{l,m}(\theta,\varphi)_p$

l	m	$\tilde{Y}_{l,m}(\theta,\varphi)_p = \tilde{\Theta}_{l,m}(\theta)\cos m\varphi$
0	0	1
1	0	$\cos\theta$
	±1	$\sin\theta\cos\varphi$
2	0	$\cos^2\theta - 1/3$
	±1	$\sin\theta\cos\theta\cos\varphi$
	±2	$\sin^2\theta\cos 2\varphi$
3	0	$\cos\theta\,(\cos^2\theta - 3/5)$
	±1	$\sin\theta\,(\cos^2\theta - 1/5)\cos\varphi$
	±2	$\sin^2\theta\cos\theta\cos 2\varphi$
	±3	$\sin^3\theta\cos 3\varphi$
4	0	$\cos^4\theta - 6/7\cos^2\theta + 3/35$
	±1	$\sin\theta\cos\theta\,(\cos^2\theta - 3/7)\cos\varphi$
	±2	$\sin^2\theta\,(\cos^2\theta - 1/7)\cos 2\varphi$
	±3	$\sin^3\theta\cos\theta\cos 3\varphi$
	±4	$\sin^4\theta\cos 4\varphi$
5	0	$\cos\theta\,(\cos^4\theta - 10/9\cos^2\theta + 5/21)$
	±1	$\sin\theta\,(\cos^4\theta - 2/3\cos^2\theta + 1/21)\cos\varphi$
	±2	$\sin^2\theta\cos\theta\,(\cos^2\theta - 1/3)\cos 2\varphi$
	±3	$\sin^3\theta\,(\cos^2\theta - 1/9)\cos 3\varphi$
	±4	$\sin^4\theta\cos\theta\cos 4\varphi$
	±5	$\sin^5\theta\cos 5\varphi$
6	0	$\cos^6\theta - 15/11\cos^4\theta + 5/11\cos^2\theta - 5/231$
	±1	$\sin\theta\cos\theta\,(\cos^4\theta - 10/11\cos^2\theta + 5/33)\cos\varphi$
	±2	$\sin^2\theta\,(\cos^4\theta - 6/11\cos^2\theta + 1/33)\cos 2\varphi$
	±3	$\sin^3\theta\cos\theta\,(\cos^2\theta - 3/11)\cos 3\varphi$
	±4	$\sin^4\theta\,(\cos^2\theta - 1/11)\cos 4\varphi$
	±5	$\sin^5\theta\cos\theta\cos 5\varphi$
	±6	$\sin^6\theta\cos 6\varphi$

Arranging atoms with the same or similar structure of outer shells one under another, in accordance with Table 3.2, we arrive at the *periodic-nonperiodic law of spherical spaces* [6] that constitutes periodic table (Table 3.3), slightly differing from the conventional one of Mendeleev.

As it turned out, the discrete model of atoms is in perfect agreement with both basic scattering experiments and most of the physics of atoms as we know it [6]. Moreover, this atomic model reveals in an understandable form various facets of different phenomena misunderstood until now. Energetic calculations, such as the bond energy in such an atom viewed as a neutron molecule, *etc.*, also confirm the consistency of this model with experimental fact [6].

4. Conclusion

The quantum mechanics foundation for the description of the atomic structure has been analyzed. Avoiding the gross errors of quantum mechanics, we have taken into account both the complex presentation of the wave function and the constant value of the wave number *k* in the wave equation. We keep in mind the origin of wave equation, before using it in quantum mechanics. The constant value of *k* follows from the wave character of space at any of its levels, including the atomic level [1]. Thus, the problem of the structure of atoms has a new and convincing solution.

The wave equation of space (3.1) takes a form similar to the basic equation of quantum mechanics (1.1), but these equations are different in contents because*:*

1) The factor k^2 in the equation of space (3.1) is the **fundamental constant** $k = \omega / c$, whereas in (1.1) it is a **variable quantity** equal to $k = \pm\sqrt{2m(W - U)/\hbar^2}$, in which $U(x, y, z) = -Ze^2 / 4\pi\varepsilon_0 r$ is the 'potential energy of the electron in the field of a nucleus, depending on electron coordinates'. The last condition is based on the nuclear model of atoms and on the supposition about the Coulomb kind of interactions between the nucleus and the electrons in atoms.

2) The wave equation of quantum mechanics (1.1) 'describes', thus, the motion of the electron around an atomic nucleus with the charge *Ze*, whereas Eq. (3.1) really describes the structure of the wave space, including the atomic space as the discontinuous part of the wave space.

Therefore, the **solutions** of the wave equation of space (3.1) and the quantum mechanics equation for electrons (1.1) *are different* because they describe different phenomena and have different meaning.

Table 3.2 Discrete structure of atomic spheres with potential nodes and rings. 'N' and 'S' denote north and south polar nodes.
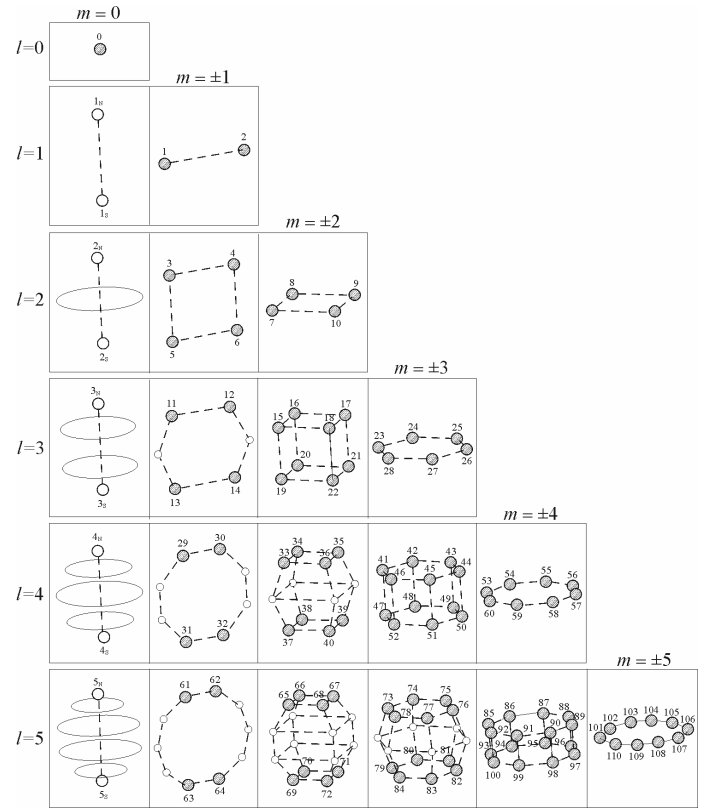
"N" and "S" are subscripts designating the "north" and "south" polar nodes (at *m* = 0).

Table 3.3. Solutions of the wave equation (3.1) presented in the traditional form of the periodic law of chemical elements; or the quasiperiodicity as a result of similarity of the structure of external shells of abstract atoms drawn in Table 3.2.

It should also be noted that quantum mechanics does not directly use its own solutions, as is done in realistic science. Instead it operates with the squared modulus of the wave function. This extraordinary illogical act allows one to drop 'an imaginary part' of the wave function, which is believed conventionally to 'have no physical meaning'. However, a tradition is not a proof. In his time, Leibniz wrote, "*A complex number is a fine and wonderful refuge of the divine spirit, as if it were an amphibian of existence and nonexistence.*" Unfortunately, creators of quantum mechanics have neglected this insight. In the past, negative numbers were also named 'imaginary', because they were considered to be unreal quantities 'smaller than nonexistence'. But in the course of time, emotions cooled and the 'imaginary' negative numbers turned into real ones. However, the square root of a negative number today remains 'imaginary'. The above-mentioned problems have been solved in works [6, 8]. Moreover, the hypothesis that considers the squared modulus of the wave function as the measure of probability is not justified.

The solutions of the wave equation of space here obtained yield the discrete (nodal) structure of matter-space, and the periodic-nonperiodic law of atomic structures. This makes possible the understanding of characteristic features of different atoms, which are (in that case) quasispherical neutron (*H*-atom) molecules of the real physical space, and their arrangement in accordance with the obtained solutions in the periodic table. Theoretical by nature, the new periodic table and Table 3.2 extend our ability to explain well known features of elements, and the structure of their compounds [9]. The data obtained make it possible to reveal, from the new point of view, the mysterious properties of atoms, including the nature of the periodicity itself.

Publications on the above-discussed theme and related to the approach accepted by the authors have not been found in literature; therefore, there are no references on similar works of others. As concerns quantum mechanical concepts considered, these can be found in the referring works or in any from thousands of widely accessible monographs and textbooks on quantum mechanics easily understandable to keen college seniors and graduate students.

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