

On the Spectral Terms of *H*-atom

Describing the atomic structure, quantum mechanics cannot do without *wave* functions. What is the reason? The fact is that interactions at the atomic and subatomic levels have the *wave nature*. Let us analyze to what results it leads, if we will consider this. We will turn to some physical processes, apparently inherent in *H*-atom (and hence, in any atom), which were not discussed until now in the framework of the standard model.

H-atom represents a dynamic paired centrally symmetric system, the proton-electron. A central spherical component (proton) has the *spherical wave field*. By this radial field, proton relates with the surrounding field-space and with an orbiting electron. The orbital motion, in turn, is associated with the *cylindrical wave field*. Hence, both dynamic components of the proton-electron system have to be described, accordingly, by spherical and cylindrical wave functions.

Electron transitions in atoms depend on the structure of their radial shells. In the *central spherical wave field* of *H*-atom, amplitude of radial oscillations of a spherical shell of the proton is

$$A_s = \frac{A\hat{e}_n(kr)}{kr}, \quad (1)$$

where $\hat{e}_n(kr) = \sqrt{\frac{\pi kr}{2}}(J_{n+\frac{1}{2}}(kr) \pm iY_{n+\frac{1}{2}}(kr))$, $k = \omega_e/c = \text{const}$ is the wave vector. At that, $J(kr)$ and $Y(kr)$ are Bessel functions; ω_e is some fundamental “carrier” frequency of the spherical wave field [1, 2], equal to the frequency of oscillations of the pulsating spherical shell of the proton. Its amplitude energy takes the following form

$$E_s = \frac{M_0\omega_e^2 A_s^2}{2} = \frac{M_0\omega_e^2}{2} \left(\frac{A}{kr}\right)^2 |\hat{e}_n(kr)|^2 = \frac{M_0 c A^2}{2r^2} |\hat{e}_n(kr)|^2, \quad (2)$$

where M_0 is the proton mass, A is the constant equal to the oscillation amplitude at the sphere of the wave radius ($kr=1$). Because $kr_1 = z_{n,1}$ and $kr_s = z_{n,s}$, where $z_{n,s}$ and $z_{n,1}$ are zeros of Bessel functions $J_{n+\frac{1}{2}}(kr)$ [3], the following relation is between radial shells: $r_s = r_1 \frac{z_{n,s}}{z_{n,1}}$. Here, the

subscript n indicates the order of Bessel functions and s , the number of the root. The last defines the number of the radial shell. Zeros of Bessel functions define the radial shells with zero values of radial displacements (oscillations), i.e., shells of stationary states. The energy E_s is the total potential-kinetic energy of oscillations.

In the *cylindrical wave field*, the energy E_f , as the sum of energies of two mutually-perpendicular potential-kinetic oscillations of the orbiting electron, is (in the simplest case) equal to

$$E_f = m_e v^2 = m_e \omega^2 A_f^2 = m_e \omega^2 \left(\frac{a}{\sqrt{kr_0}}\right)^2 = 2\pi m_e \nu A_f \nu, \quad (3)$$

where m_e is the mass of the electron; r_0 is the radius of its orbit; ν is the frequency and $\nu = \omega A_f$ is the amplitude speed of its oscillations; $A_f = \frac{a}{\sqrt{kr_0}}$ is the amplitude of its oscillations.

Because $k = \omega/c$ (in Eq. (3)), hence $E_f = h\nu$, where $h = \frac{2\pi m_e c a^2}{r} = 2\pi m_e \nu A_f$ is an elementary action. The constant a , equal to the oscillation amplitude at the Bohr orbit r_0 with the length in one wave, is

$$a = \sqrt{\frac{hr_0}{2\pi m_e c}} = 4.52050647 \cdot 10^{-10} \text{ cm}, \quad (4)$$

where $h = 2\pi m_e v_0 r_0 = 6.626176 \cdot 10^{-27} \text{ erg} \cdot \text{s}$ is Planck constant. If $kr_0 = \frac{\omega r_0}{c} = \frac{v_0}{c}$, where v_0 is the Bohr speed, then the amplitude of oscillations is equal to the Bohr radius: $A_f = \frac{a}{\sqrt{kr_0}} = r_0$.

In a case when exchange (interaction) between spherical and cylindrical fields takes place, the equality $E_f = \Delta E_s$ is valid. Consequently, we have

$$h\nu = \frac{M_0 c^2 A^2}{2r_0^2} \left(\frac{|\hat{e}_p(kr_m)|^2 z_{p,1}^2}{z_{p,m}^2} - \frac{|\hat{e}_q(kr_n)|^2 z_{q,1}^2}{z_{q,n}^2} \right) \quad (5)$$

Since quantum numbers in the spectral formula (5) are roots of Bessel functions, we can regard these as a mathematical variant of spectral terms. Exchange of energy between the proton and the electron in real conditions occurs on the background of exchange with the surrounding H -atom fields-spaces of a different nature. Hence, the equation of exchange (interaction) should be presented as $E_f = \Delta E_s + \delta E$, where δE takes into account the factor of external influences (perturbations).

At $p=q=0$, zeros of Bessel functions $J_{0+\frac{1}{2}}(z_{0,s})$ are equal to $z_{0,s} = s\pi$ and $|\hat{e}_0(kr_s)|^2 = 1$; then the equation (5) is transformed into the spectral formula for H -atom:

$$\frac{1}{\lambda} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right), \quad (6)$$

where $R = \frac{M_0 c A^2}{2hr_0^2}$ is the Rydberg constant.

Because $R = \frac{R_\infty}{1+m_e/M_0} = 1.09677583 \cdot 10^5 \text{ cm}^{-1}$, hence $A = r_0 \sqrt{\frac{2hR}{M_0 c}} = 9.009369379 \cdot 10^{-13} \text{ cm}$.

Assuming in the formula (1) that kr is equal to the first extremum of the spherical function of the zero order, unequal to zero ($kr = 4.49340946$), we find the first maximal amplitude of radial oscillations:

$$\langle A_s \rangle = \frac{1}{\sqrt{2}} \frac{A}{kr} = 1.417762222 \cdot 10^{-13} \text{ cm}. \quad (7)$$

The center of masses of the proton, performing such oscillations, forms a **dynamic spherical volume** with the radius equal to the amplitude of the oscillations and its volume can be regarded as some **nucleus**.

- [1]. L.G. Kreidik and G.P. Shpenkov, *Alternative Picture of the World*, Vol. 1-3, Bydgoszcz, 1996.
- [2]. L.G. Kreidik and G.P. Shpenkov, *Foundations of Physics; 13.644...Collected Papers*, Bydgoszcz, 1998.
- [3]. F.W.J. Olver, ed., *Royal Society Mathematical Tables*, Vol. 7, *Bessel Functions*, part III, *Zeros and Associated Values*, Cambridge, 1960.