

Three-dimensional solutions of the Helmholtz equation

George Shpenkov

<http://shpenkov.janmax.com/talk2015Gottingen.pdf>

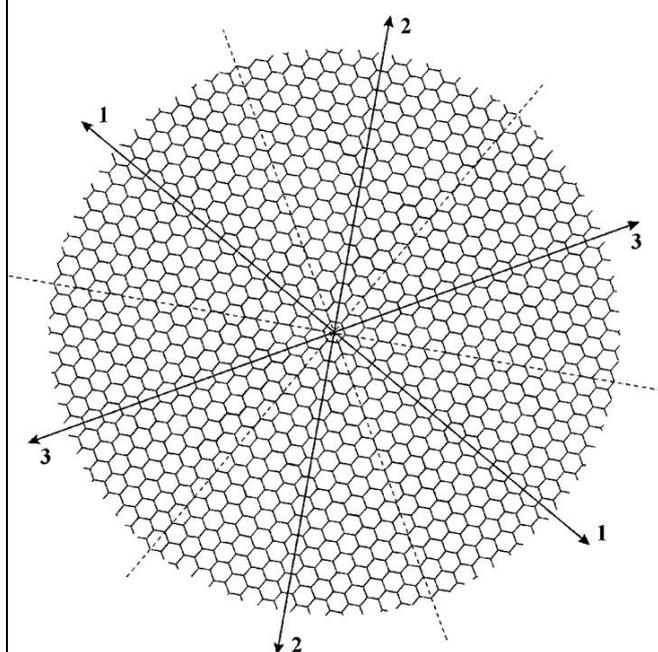
Report

16 March 2015

What relation has the Helmholtz equation to crystallography, crystal chemistry and, generally, to atomic structure? It is the subject of my talk.

Picture 2. As an example let us consider one of the allotropes of carbon – **graphene**. It has **two-dimensional hexagonal lattice** and, as commonly believe, **six-fold** rotational symmetry.

The structure of Graphene



A direct image of a single-layer graphene sheet



[J.C. Meyer, C. Kisielowski, R. Erni, M.D. Rossel, M.F. Crommie, and A. Zettl. "Direct Imaging of Lattice Atoms and Topological Defects in Graphene Membranes", NANO LETTERS 2008, Vol. 8, No. 11, 3582-3586]

Therefore, directions 1-1, 2-2, and 3-3 depicted here are crystallographically identical, and, in full agreement with the basic symmetry theory, all properties along them must be the same, for example, electronic conductivity.

However, our studies show that this is not true.

As it turned out, if one takes into account an invisible part (that we have found) in the structure of carbon atoms (graphene constituents), graphene has only two-fold rotational symmetry.

This follows from particular solutions of the Helmholtz equation.

And the structure and, hence, properties along one of the indicated here three directions differ from another two. Conducted tests confirmed this feature predicted theoretically.

How we have come to this discovery?

Picture 3. Most physicists understand that the SM exhausted itself long ago and holds back development of physics. Actually,

**Due to dominance of the
Standard Model**

**Modern physics is unable to comprehend till now
such fundamental things as the:**

- nature of charges,
- origin of mass,
- nature of gravitation.

A great mystery is still the physical meaning of the:

- speed* of light c in a *rest* energy formula, $E_0 = m_0 c^2$
- fine structure constant α .

It is impossible on its base to derive the:

- structure and relative atomic masses of isotopes,
- magnetic moments of neutron and proton;

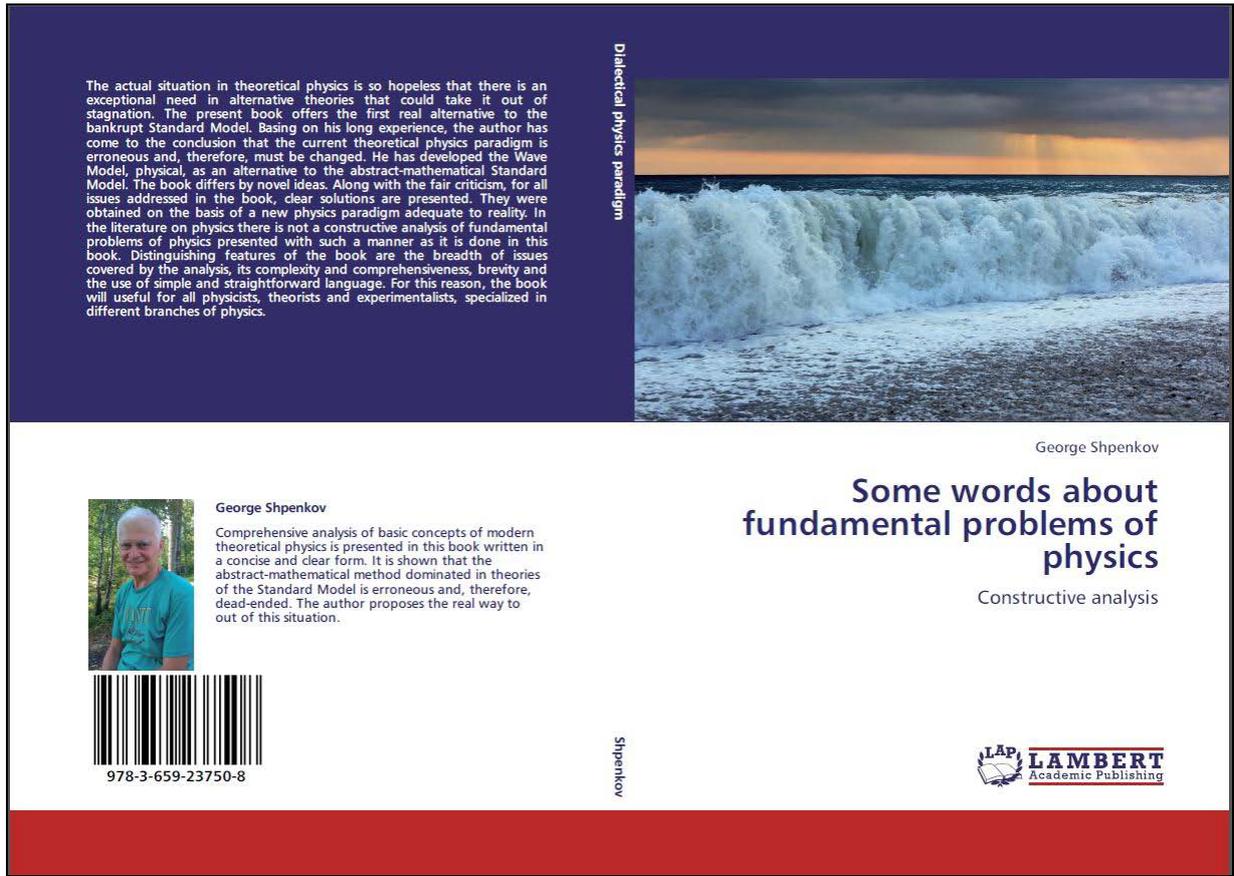
and to build a

- unified field theory; and so on.

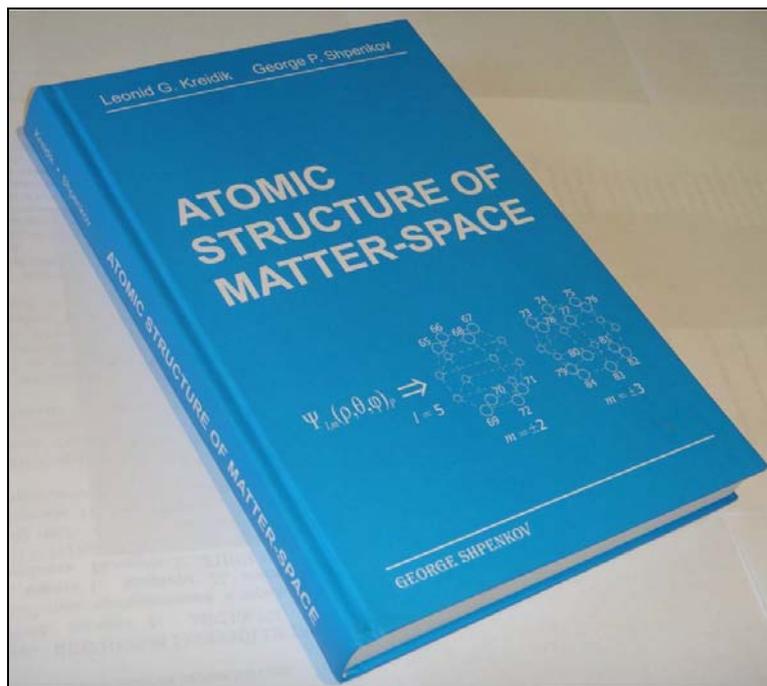
All attempts undertaken to improve the SM, for example, with help of string theories, turned up unsuccessful.

We have chosen essentially other way. During more than two decades, we develop conceptually a new basic physics theory, which can be considered, judging by the results, as an alternative to the Standard Model.

Picture 4. Short articles devoted to our studies are in a **book** shown here.



Picture 5. Basic details of a new theory, that we develop, one can find in a book “Atomic Structure of Matter-Space” published in 2001.



Picture 6. A new physics paradigm, to which we adhere.

A new physics paradigm

is based on:

(1) Dialectical philosophy and dialectical logic

(2) A postulate on the wave nature of all phenomena and objects in the Universe

Picture 7. Following the postulate.

Following the postulate

*The wave structure of matter-space is described by the well-developed methods of classical wave physics, in particular, by the **general wave equation***

$$\Delta \hat{\Psi} - \frac{1}{c^2} \frac{\partial^2 \hat{\Psi}}{\partial t^2} = 0 \quad (1)$$

It admits the **particular solution** of the form

$$\hat{\Psi}(\rho, \theta, \varphi; t) = \hat{\psi}(\rho, \theta, \varphi) e^{\pm i\omega t} = \hat{R}_l(\rho) \Theta_{l,m}(\theta) \hat{\Phi}_m(\varphi) e^{\pm i\omega t} \quad (2)$$

which describes the **spherical standing waves**.

*The **spatial factor** $\hat{\psi}(\rho, \theta, \varphi)$ **defines the standing wave structure** and, as we have disclosed,*

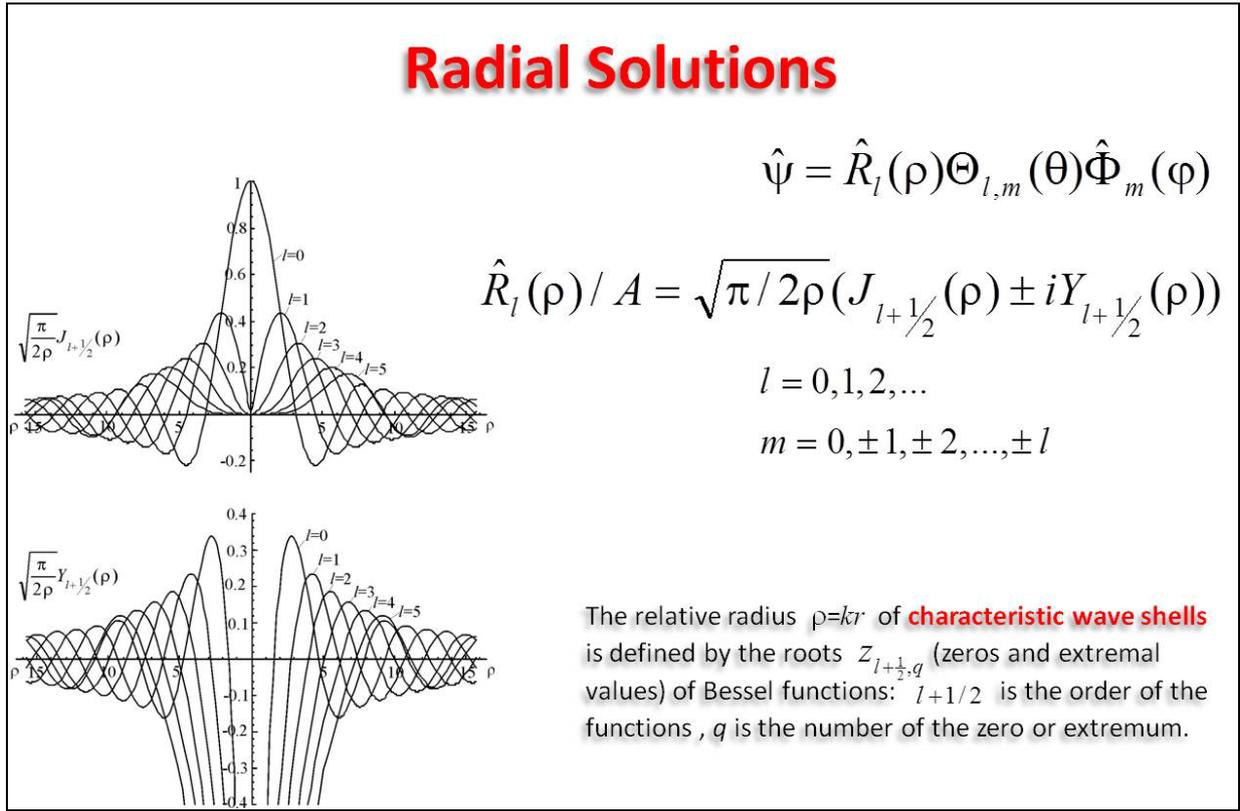
Atomic Structure

$\hat{\psi}$ is the solution of the time-independent form of Eq. (1), the **Helmholtz equation**,

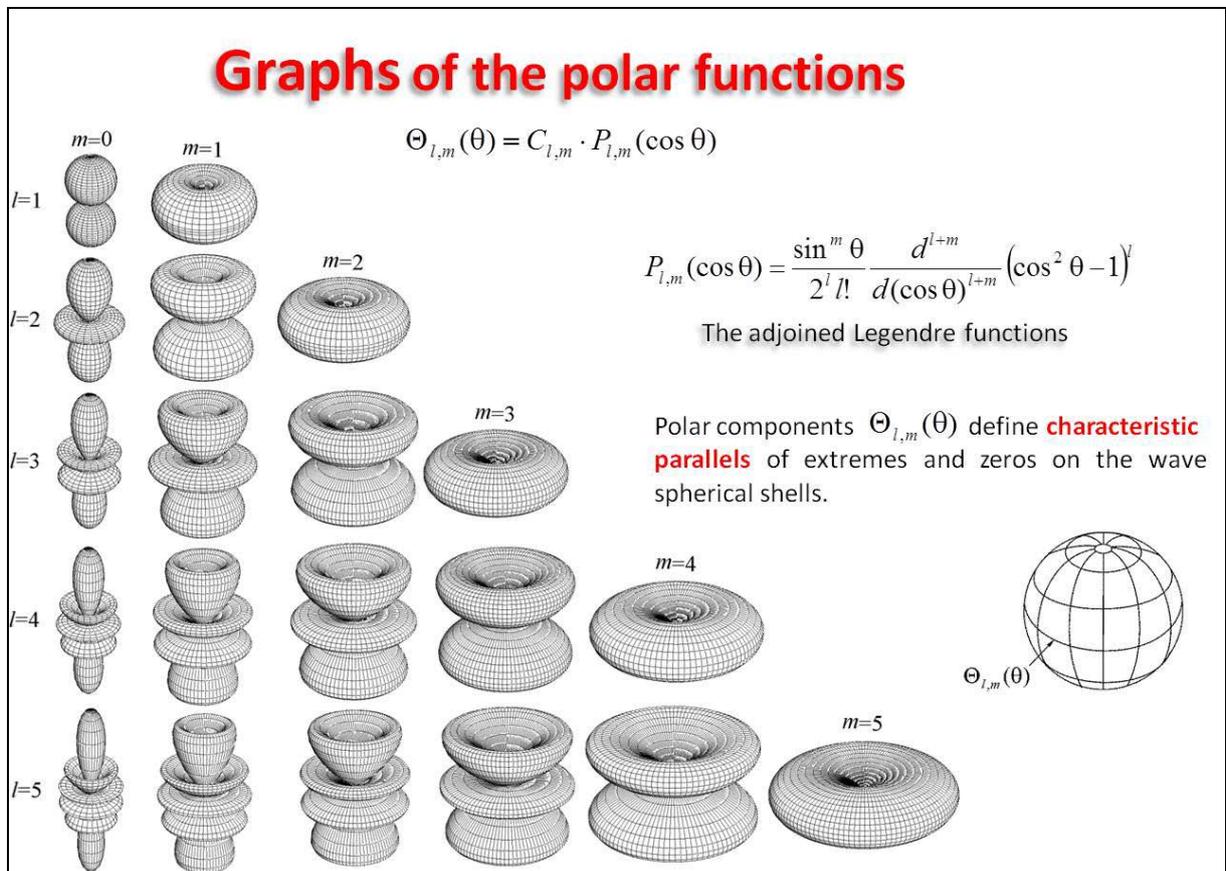
$$\Delta \hat{\psi} + k^2 \hat{\psi} = 0 \quad (3)$$

*(The wave vector k is the constant. At the **atomic** and **subatomic** levels $k = \omega_e / c$, where ω_e is the **fundamental frequency** of the levels; $\rho = kr$)*

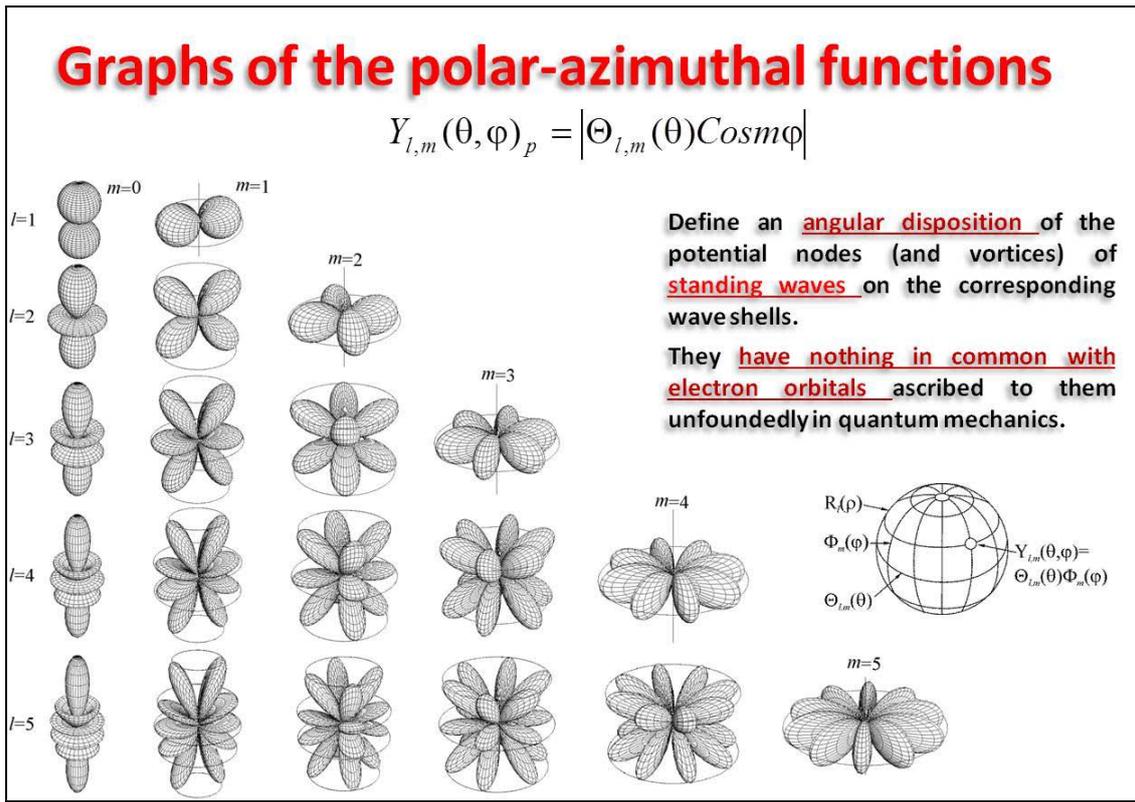
Picture 8. Radial solutions.



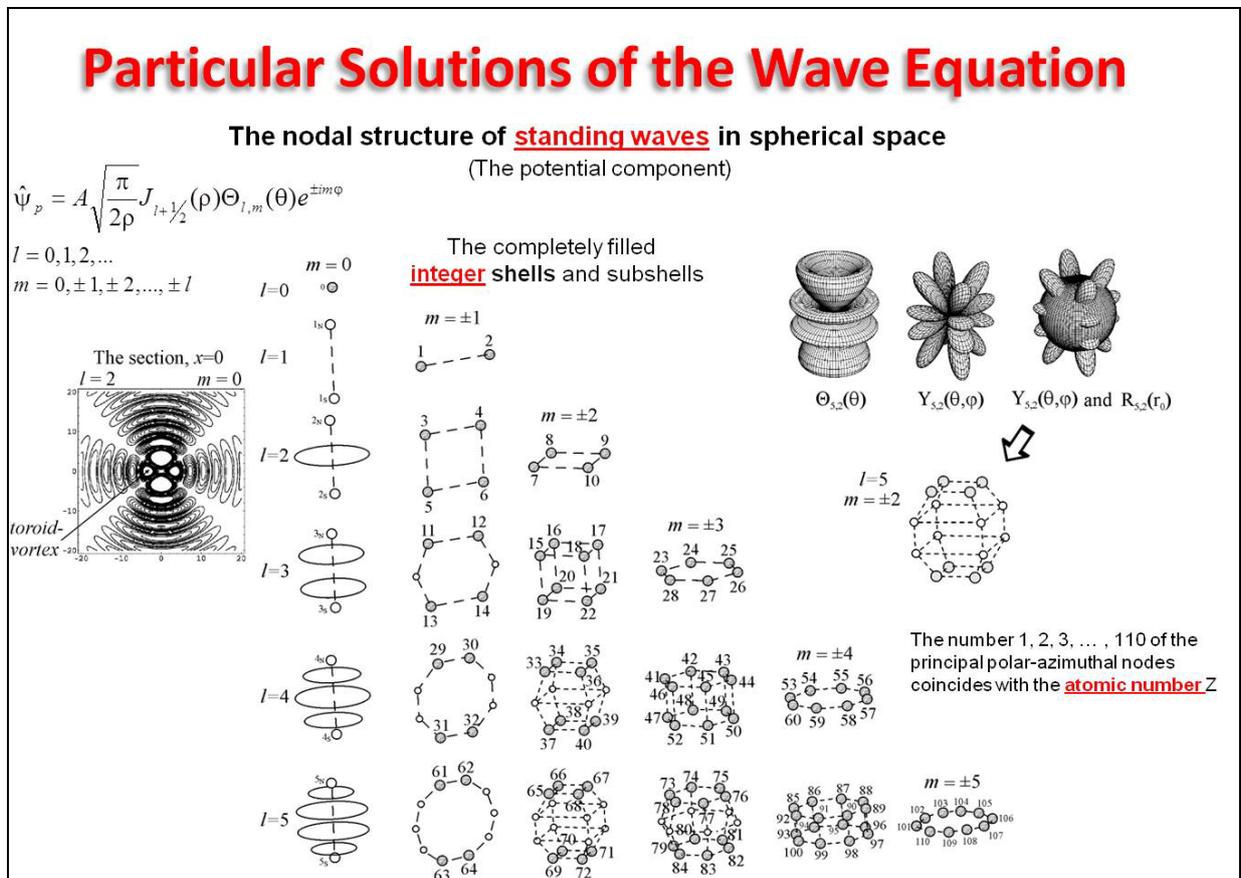
Picture 9. Polar solutions.



Picture 10. Polar-azimuthal solutions.



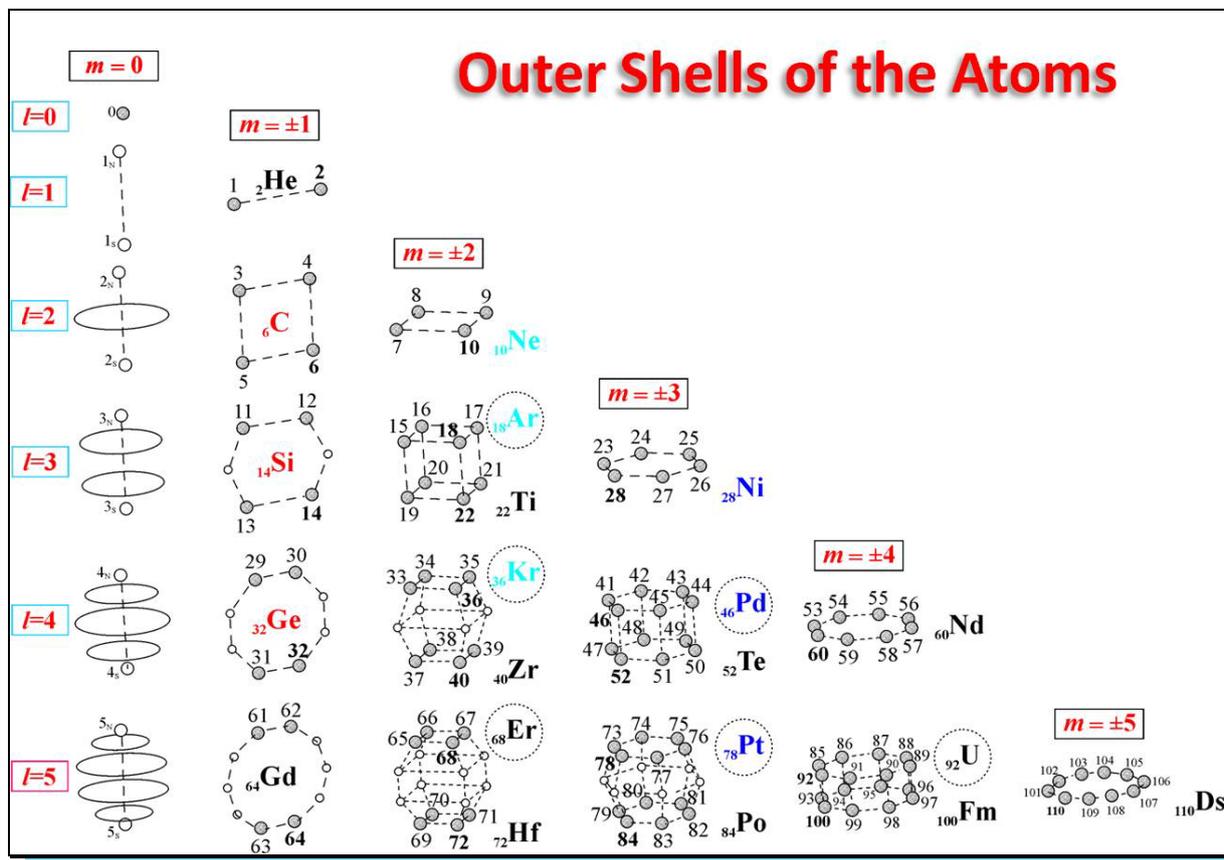
Picture 11. As a result, we arrive at the nodal structure of standing waves, which we present in the form shown here.



It is a key form of the solutions that led us, after the comprehensive analysis, to the conclusion that these solutions define the shell-nodal structure of the atoms.

And the ordinal numbers, which we have ascribed to the principal polar-azimuthal nodes, correspond to the atomic numbers Z .

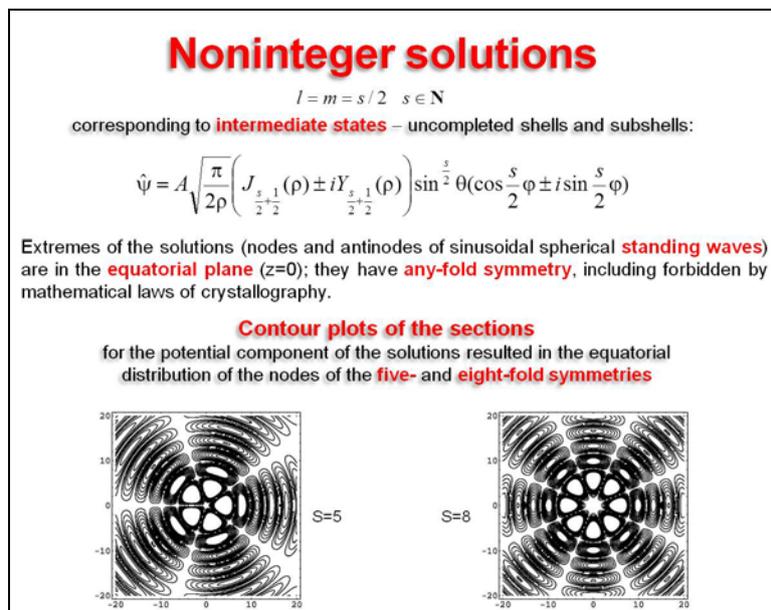
Picture 12. The atoms at the corresponding to them outer shells, integer and half-integer, are indicated here. The atoms having uncompleted outer shells (Ar, Kr, Er, Pd, Pt, U) are shown in dashed circles.



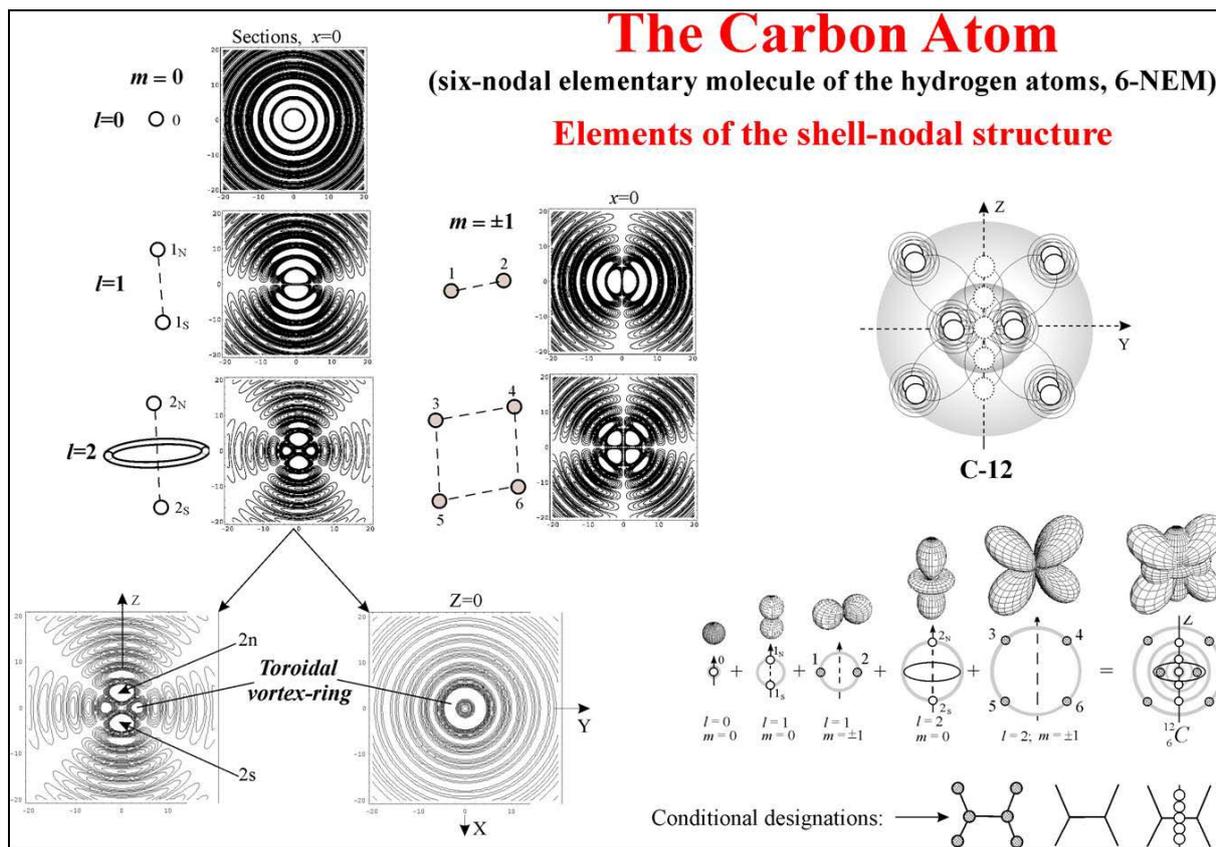
One should pay attention to the clearly seen **regularity** and **quasi-periodicity** as well as to the presence of the **collateral** (unnumbered) **nodes** in external shells of the atoms of classical **semiconductors**, Si and Ge.

Picture 13. Half-integer and, in general, **noninteger solutions** relate to intermediate shell-states. They define **uncompleted** external shells inherent in such atoms (see Picture 12) as, for example (for $l = 2, m = \pm 1$) ${}^3\text{Li}$ (having one node under the number 3 in its outer shell), ${}^4\text{Be}$ (with two external nodes, the numbers 3 and 4), ${}^5\text{B}$ (with three external nodes, the numbers 3, 4, and 5) or (for $l = 2, m = \pm 2$) ${}^7\text{N}$ (having one node under the number 7 in its outer shell), ${}^8\text{O}$ (two nodes, 7 and 8), ${}^9\text{F}$ (three nodes, 7, 8, and 9), and *etc.*

These **uncompleted shells** are **in the equatorial plane** and have **any-fold** symmetries, including forbidden by mathematical laws of crystallography, dependent on the number of nodes in the shells. So the external shells of ${}_{18}\text{Ar}$, ${}_{36}\text{Kr}$, and ${}_{68}\text{Er}$ have the same number of nodes in external uncompleted shells, lying in the equatorial plane, as ${}_{10}\text{Ne}$.



Picture 14. Elements of the shell-nodal structure of the **carbon atom** are presented in this Picture not only **schematically**, but also there are shown the **strict graphical solutions** for their sections in the indicated planes. An interesting element of the carbon atom structure (and not only carbon) is a standing wave **toroidal ring-vortex** at the shell related to the solution for $l = 2, m = 0$.



At the bottom right there are shown **conditional designations** of the carbon atom, which is actually the **six-nodal elementary molecule** of the **hydrogen atoms** to which we refer **proton, neutron, and protium**.

Picture 15. The 6-nodal elementary molecule as a whole, in three projections, is shown in this Picture. Each of its six **potential** nodes is filled with **coupled** hydrogen atoms. Internodal distances are defined by the roots of Bessel functions – solutions of the radial part of the wave equation.

The Carbon Atom

(Six-Nodal Elementary Molecule of the Hydrogen Atoms, 6-NEM)

An elementary particle and an atom are the spherical formations pulsating at the frequency ω_e

Fundamental frequency and the **fundamental wave radius** of the atomic and subatomic levels:

$$\omega_e = 1.869162559 \times 10^{18} \text{ s}^{-1}$$

$$\lambda_e = \frac{c}{\omega_e} = 1.603886492 \times 10^{-8} \text{ cm}$$

Fundamental frequency and the **fundamental wave radius** of the gravitational level:

$$\omega_g = 9.158082264 \times 10^{-4} \text{ s}^{-1}$$

$$\lambda_g = \frac{c}{\omega_g} = 327.4 \times 10^6 \text{ km}$$

$E = \omega_e^2 \frac{m_1 m_2}{8\pi \epsilon_0 r}$ **Binding energy** of the nodes in **6-NEM** ($^{12}_6\text{C}^{-4}$):

$E_{C,ion} = 92.34932212 \text{ MeV}$

$\omega_e m_2 = q_1$ and $\omega_e m_1 = q_1$ are **exchange charges** of interacting nucleons

$\epsilon_0 = 1 \text{ g} \cdot \text{cm}^{-3}$ is the **absolute unit density**

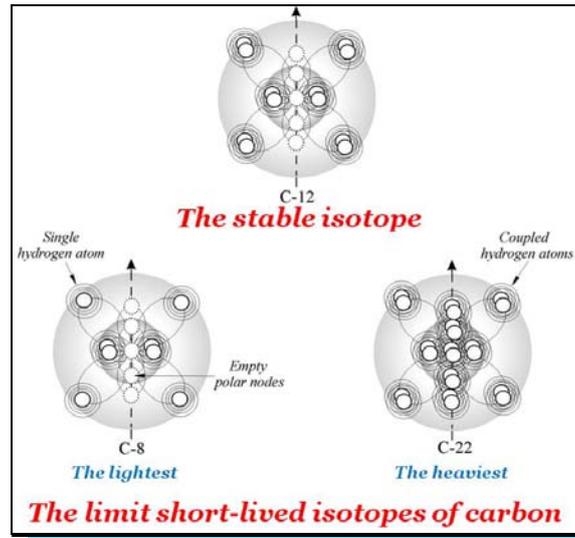
Internodal **bonds are strong**; they are defined by the formula indicated at the bottom left. As follows from calculations, **binding energy** of all nodes of the carbon atom is equal to 92.3 MэВ. This value practically coincides with the literature data for the binding energy of the carbon nucleus. The formula of binding energy E originates also from the Wave Model.

In the formula of binding energy, it enters the **fundamental frequency**, ω_e , determining **interactions at the atomic and subatomic levels**, setting the strictly defined **internodal distances**. Note that the **fundamental radius** equal to the ratio of the speed of light c and the fundamental frequency ω_e is about $1.6 \times 10^{-8} \text{ cm}$. Its double value, the fundamental diameter ($3.2 \times 10^{-8} \text{ cm}$), coincides with an average value of lattice parameters.

Kinetic nodes (antinodes of standing waves), being the places of intensive motion, are empty. **Polar potential-kinetic nodes** located along the z -axis – places of rest and motion simultaneously – also empty in a stable carbon atom C-12.

Picture 16. **Isotopes** of carbon, as any atom, are defined by the extent of filling the following nodes: **potential** polar-azimuthal and **potential-kinetic** polar, at which it is preserved (even for a moment) the specific nodal structure of its spherical shells, inherent just in the carbon atom distinctive it from other atoms.

As follows from the shell-nodal structure of carbon, the possible limit short-lived, lightest and heaviest, carbon isotopes are C-8 and C-22. Actually, experimental data confirm this.



Picture 17. In full agreement with the regularities and quasiperiodicity found in the shell-nodal structure of the atoms, I built the **Periodic Table of the Atoms**, which in fact is the **Periodic Table of Elementary Molecules of Hydrogen Atoms**. Atoms, as elementary molecules, are grouped in the Table according to the number of principal potential polar-azimuthal nodes in their outer wave shells.

PERIODIC TABLE OF THE ATOMS

(Z-Nodal Elementary Molecules of the Hydrogen Atoms, Z-NEMs)
 $Z=N = \{1, 2, 3, 4, \dots\}$

Solutions of the wave equation $\Delta\psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$
 The atoms grouped according to the **number N** of principal potential polar-azimuthal nodes in their wave shells

The number $N \{1, 2, 3, \dots, 110\}$ of the node coincides with the atomic number Z

Unnumbered *collateral* polar-azimuth nodes

□ • unstable elements

Atoms with completely filled external wave shells

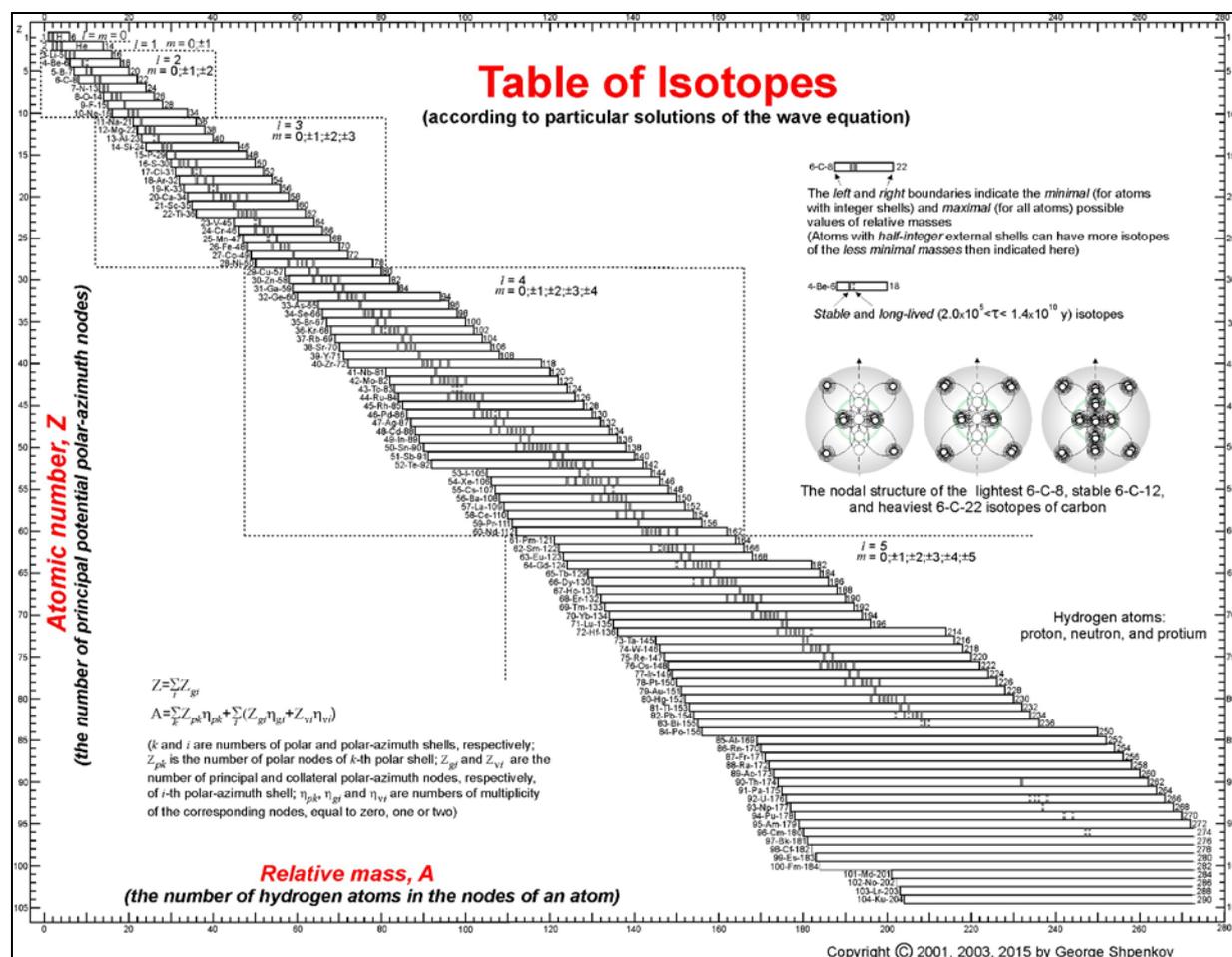
2He			
10Ne			
28Ni			
60Nd			
100Fm			

Polar nodes and toroids-vortices

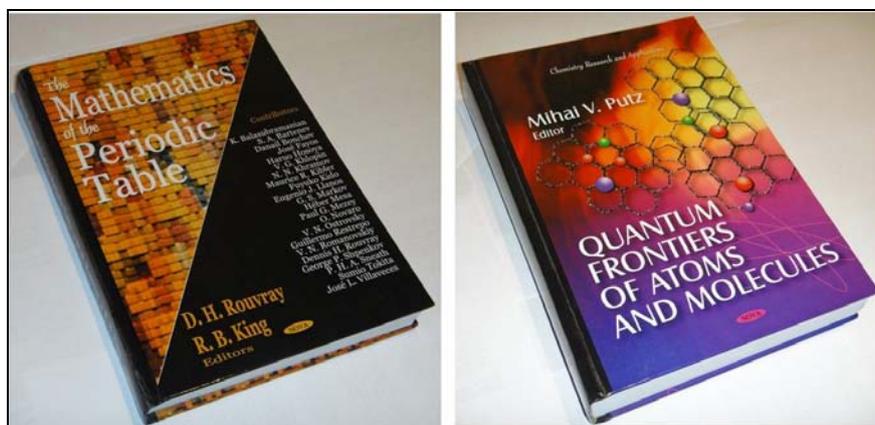
The hydrogen atoms: proton, neutron, and protium

[1] *Alternative Picture of the World*, V. 1-3, (1996); [2] *Foundations of Physics*, (1998); [3] *Atomic Structure of Matter-Space*, (2001); Geo. S., Bydgoszcz by L. Kreidik and G. Shpenkov. The Shell Structure of Matter Spaces, <http://shpenkov.janmax.com/ShellStr.pdf> Copyright © 2001, 2015 George P. Shpenkov

Picture 18. The Table of all possible atomic isotopes I built on the basis of extent of filling the corresponding nodes (including collateral) of the atoms admitted by the shell-nodal structure of individual atoms, as it was done on an example of filling the nodes in carbon isotopes (Picture 16). The left edge of the graph corresponds to the limit possible lightest short-lived isotopes, the right edge – to the limit possible heaviest short-lived isotopes.

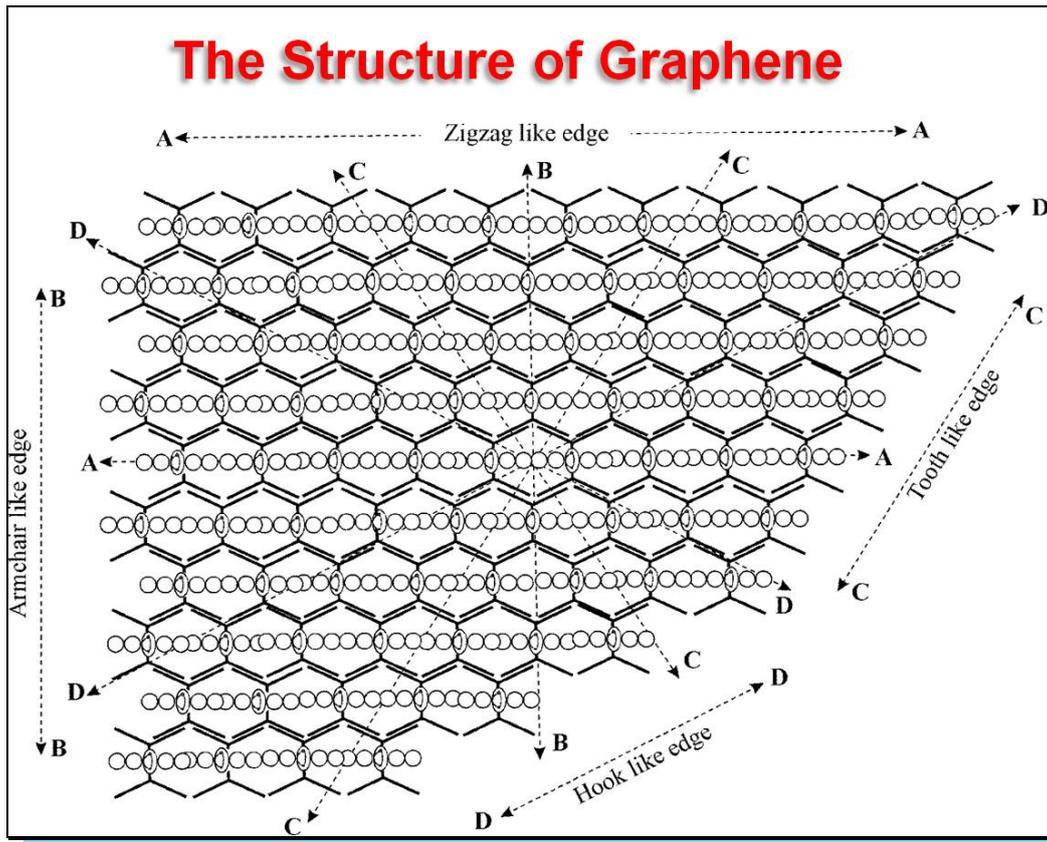


Picture 19. All details concerning the Periodic Table, built on the basis of particular solutions of the wave equation, one can find in a book “**The Mathematics of the Periodic Table**”.



In a book “**Quantum Frontiers of Atoms and Molecules**”, there is detail information about the **Table of Isotopes** presented in Picture 18 and about the mechanism of the formation of carbon compounds in the light of the shell-nodal structure of the carbon atom.

Picture 20. The two-dimensional hexagonal lattice of graphene is shown schematically in this Picture.



As can be seen, **interatomic bonds** are realized along the lines of strong internodal bindings inherent in each of the nearest atoms that provides in a result to the **three-multiple overlapping in a nodal point** of the crystal of the nodes belonging to three different atoms.

Here there are some peculiarities to which one needs to turn your attention. They are related to the fact that if we recognize the well-known lattice parameters of graphite, and, hence, graphene, as corresponding to reality, *i.e.*, absolutely true, then the elementary units of the graphene hexagonal lattice are carbon dimers, but not carbon atoms. Their conditional designation is the same as for carbon atoms [13]. Hence, it will take place the three-dimensional overlapping of the nodes of dimers in the nodal points of graphene crystal.

This feature is discussed in detail in the indicated above books and references presented at the end of my presentation. Because of lack of time, I will not concentrated on details here.

An ordered bond of elementary constituents, leading to the formation of graphene, is realized in such a way that a continuous **chain of empty polar nodes** is formed. The latter represents in result something like a **tunnel** along which charge carriers can move without obstacles and, hence, scattering.

Because of specific spatial structure of carbon atoms (which, as was revealed, **remind molecules**) and their specifically ordered bindings in hexagonal lattice, graphene behaves as **anisotropic crystal**; has **exotic electronic properties** and, hence, the great **potential** for practical applications. Moreover, **it provides the unique possibility** for the test on the validity of different theoretical models, theories, hypotheses, **which I took advantage.**

Picture 21. This discovery explains many interesting features of graphene and, in particular, the facts that ...”Graphene ... is an interesting mix of a **semiconductor** ... and a **metal** ...”; and that “The electrons in graphene ... have **very long mean free paths**”, etc.

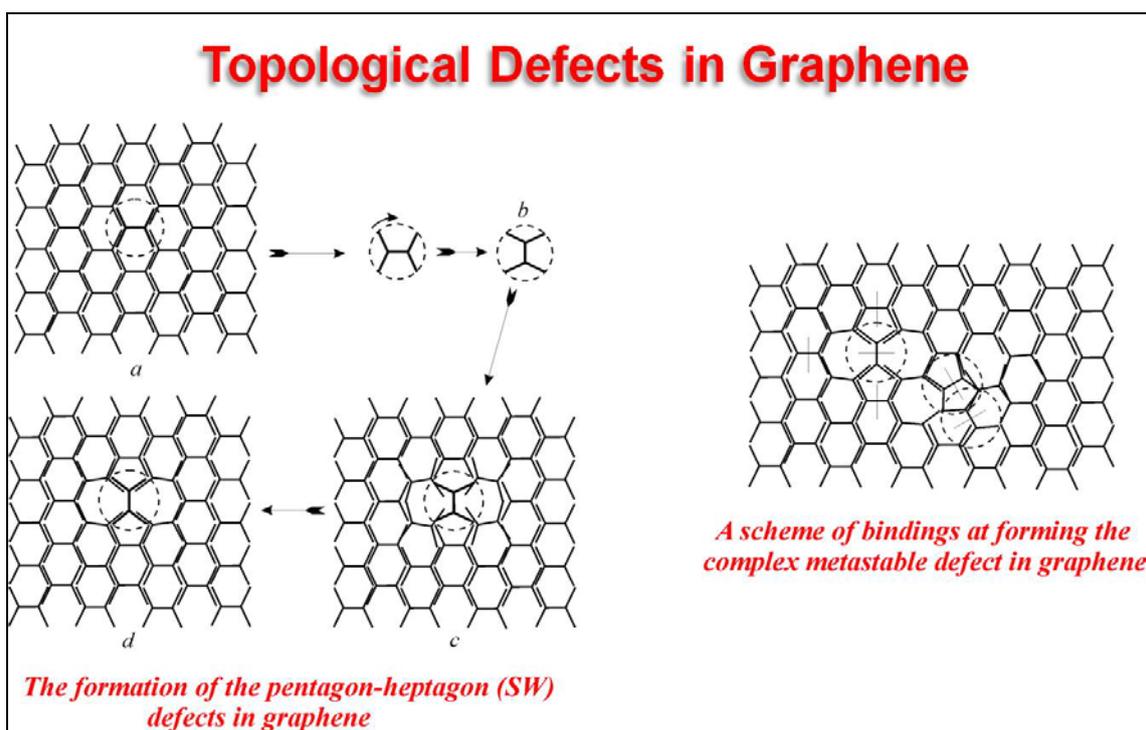
“**Graphene** ... is an interesting mix of a **semiconductor** (zero density of states) and a **metal** (gaplessness)...”

“The electrons in graphene seems to be almost insensitive to disorder and electron-electron interactions and **have very long mean free paths**” (“... reaching up to **1 μm** in the existing samples”).

“Understanding the nature ... is a problem of fundamental importance for applications”

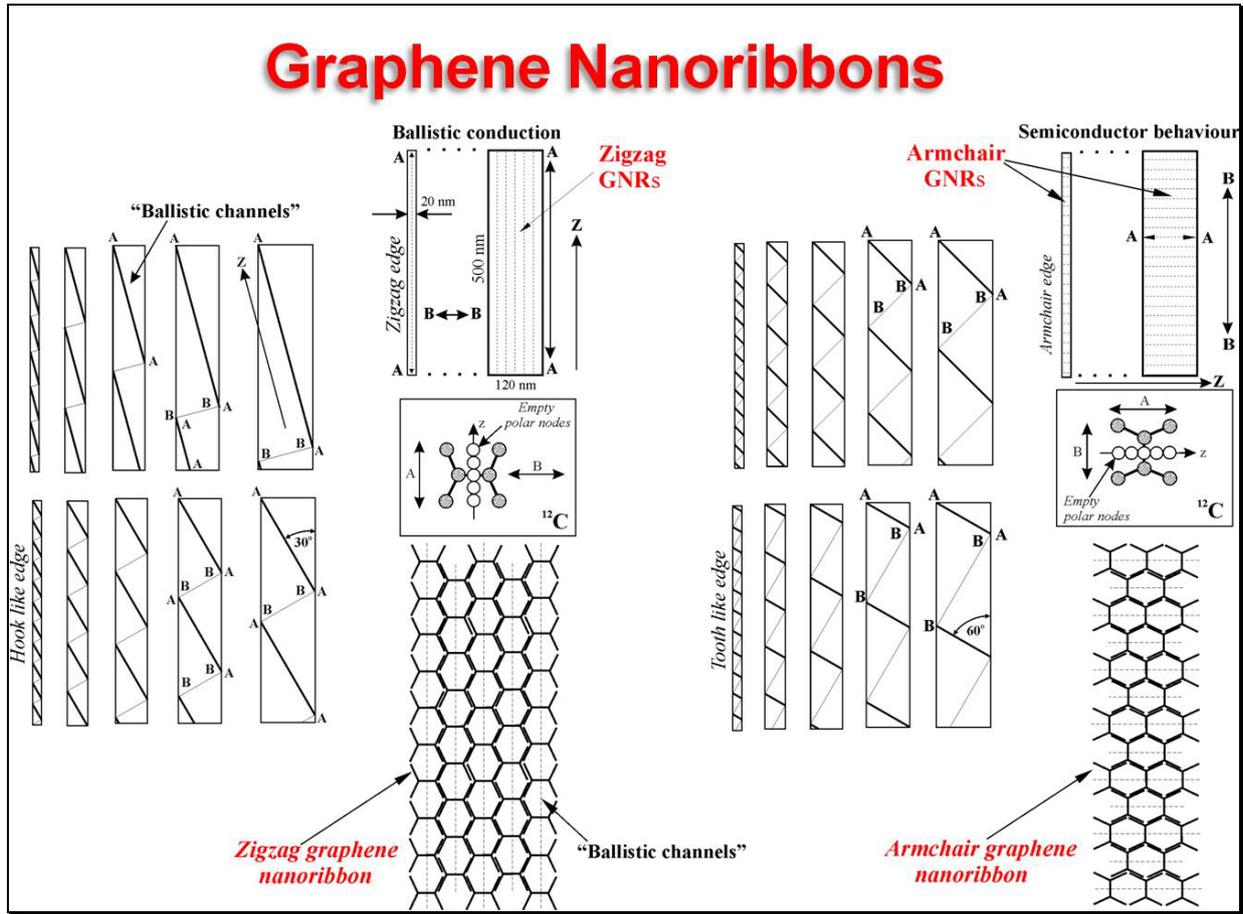
[A.H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov and A. K. Geim, *The electronic properties of graphene*, Rev. Mod. Phys. **81**, 2009]

Picture 22. Mechanism of the formation of **structural defects in graphene** is now simply explained. For example, a widely-known pentagon-heptagon (5-7) (SW) topological defect arises due to appearance when heated the rotational modes (additional to vibrational) resulted in violation of equilibrium bindings and the formation of short-lived bonds of nearest carbon atoms as shown in this figure. When cooling it takes place self-organization (self-assembly), the return of the exited region of the lattice in the equilibrium state, the defect disappears.

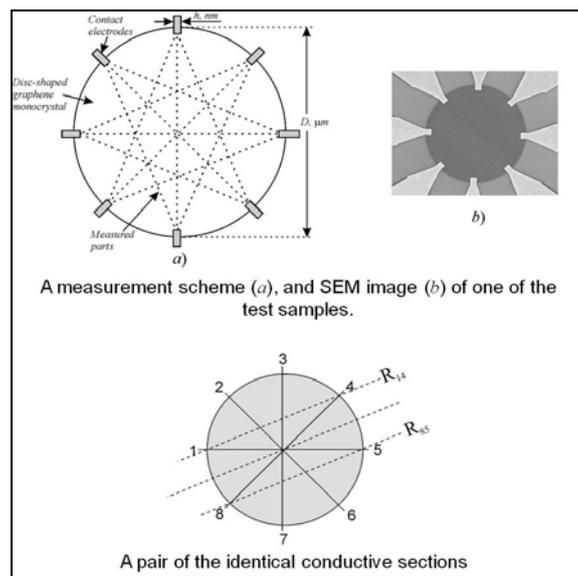


Picture 23. Graphene strips (nanoribbons, GNRs) having the different orientations (A-A) of the Z-axis (“ballistic channels”) in respect to their edges and different widths are shown here. Properties of **graphene nanoribbons**, in particular, dependence of resistance on their width and length is not related to the shape of the edges of the tapes, but **depends on the orientation of the polar empty nodes** therein which form something like a channel for the “ballistic” motion of charge carriers.

Graphene Nanoribbons



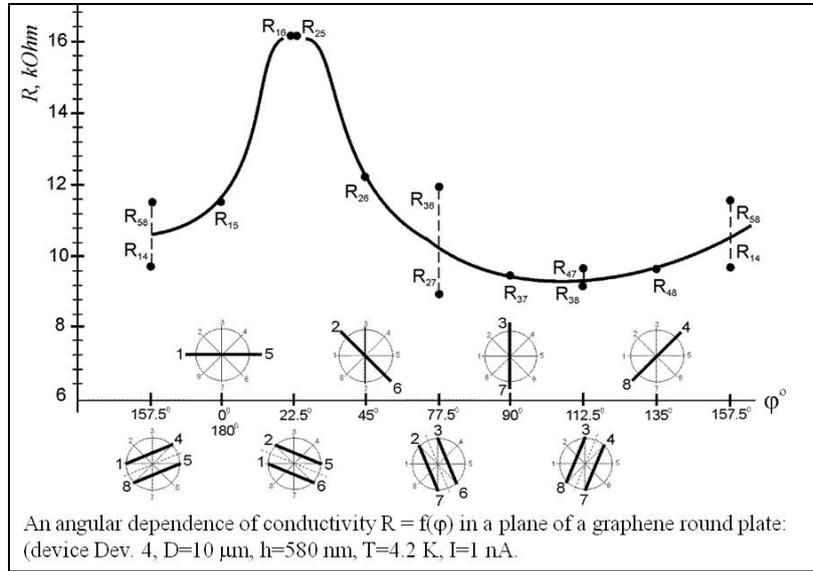
Picture 24. The experimental procedure proposed by me and realized on my initiative for the test on the existence of the **conductivity anisotropy** in graphene (caused by the presence of "the ballistic channel" as followed from the solutions of the wave equation) is clear, I assume, without comment from the **conductivity measurement circuit** shown in this Picture.



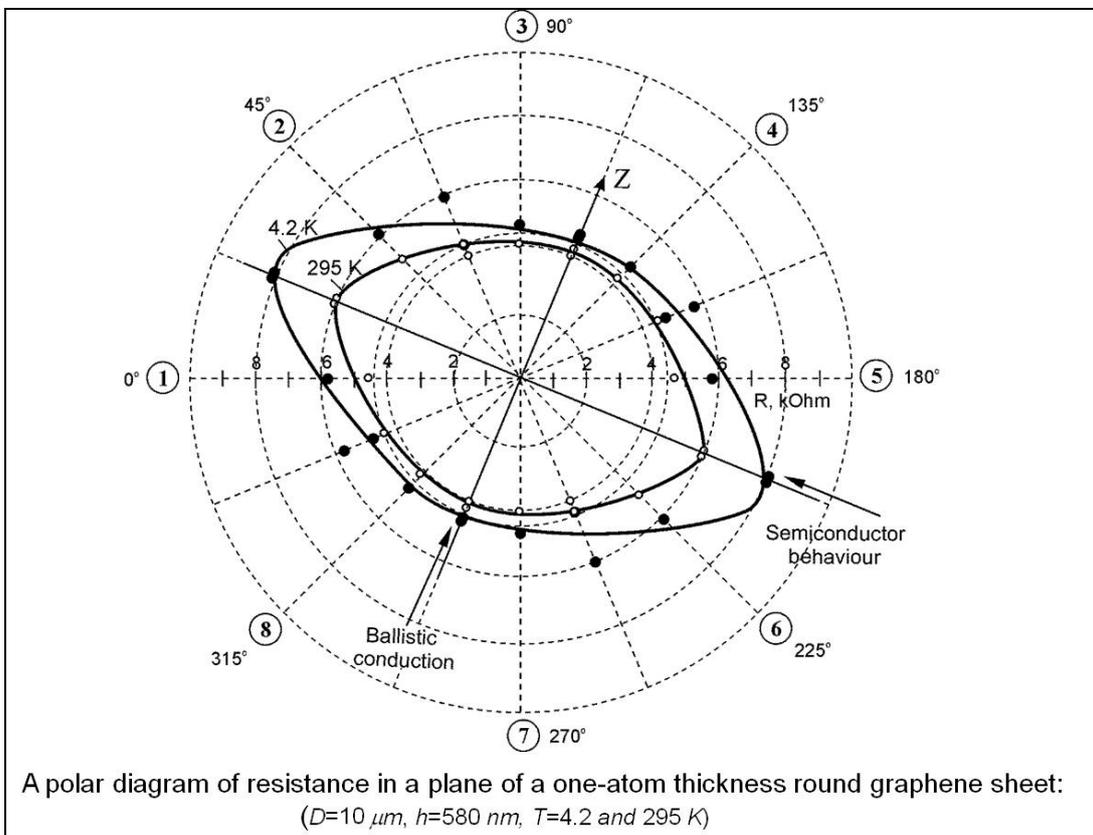
Both resistances in a **parallel pair** of identical conductive paths of the same geometrical configuration **must be equal** in value (within error margins). This allows providing an instant control of the quality of electric contacts made to the disc-shaped graphene plates, directly

during measuring the resistances. Note in this connection that the accepted measurement scheme can be used also *for verification on applicability* of the developing technological means for the creation of the perfect electric contacts on graphene.

Picture 25. The **angular dependence of the conductivity** of one of the test samples is presented here in the form of the regular graphic.



Picture 26. A **polar diagram of resistance** in a plane of a one-atom thickness round graphene sheet (diameter $D=10 \text{ mm}$; contact width $h=580 \text{ nm}$; temperature $T = 4.2$ and 295 K) is typical for all tested samples.



The **temperature dependence** of graphene conductivity along one of the directions in a plane justifies that graphene in this direction **behaves as a semiconductor**.

In a perpendicular to the latter direction, resistance in graphene practically does not depend on temperature. Obviously, this direction in graphene crystal coincides with the z-axis and, hence, with the “ballistic channel”; graphene **behaves as a metal**. Comparison with a metal concerns only the temperature dependence because the mechanisms of conductivity in a metal and in graphene differ in principle.

Two-fold symmetry of unstrained pristine graphene, originated from solutions of the Helmholtz equation, is confirmed also in independent experiments with use of nondestructive optical methods. In particular, there is information (not yet published) about observation in graphene the strong in-plane anisotropy with a periodicity of 180 degree in a visible spectral region using the Microscopic Reflection Difference Spectroscopy.

My analysis of the paper published in Phys. Rev. in 2009 [T. M. G. Mohiuddin et al., *Uniaxial strain in graphene by Raman spectroscopy: G peak splitting, Grüneisen parameters, and sample orientation*, Phys. Rev. B 79, 205433 (8 pages), 2009] has shown that the authors (not understanding this) have defined actually the orientation of characteristic crystallographic axes on a tested graphene monolayer, confirming thus (but unknowing about this) that graphene is anisotropic. I analyzed this in [13, see Picture 44 there].

Picture 27. Predicted and experimentally confirmed **anisotropy of graphene** has **practical significance** both for current laboratory studies and for the future use of graphene in electronic industry along with silicon, germanium, and other semiconductors.

It is clear; if a crystal is anisotropic then first of all it must be oriented on substrates before studying or manufacture of electronic devices. In opposite case, the uncontrolled parameters spread and properties will occur.

Practical Significance of the Tests Results

The observed anisotropy should be taken into account in the electronics industry based on use of graphene to exclude crystallographically arbitrary orientation of single-layer graphene sheets on substrates.

It is a mandatory self-evident technological operation, which provides identical in parameters electronic devices produced on monocrystals.

[G. Shpenkov, *Method for manufacturing nano electronic devices made from 2D carbon crystals like graphene and devices obtained with this method*. Int. Appl. No.: PCT/EP2010/052298; Publ. No.: WO/2010/097393;
<https://register.epo.org/application?number=EP10707863&lng=en&tab=main>]

Picture 28

A Particular Conclusion

Graphene has two-fold rotational symmetry and is an anisotropic crystal.

The invisible polar nodes form a **channel** favourable for ballistic motion of electrons.

Hence, it becomes understandable and explainable now:

Why *"Graphene...is an interesting mix of a semiconductor...and a metal"*.

Why *"Electrons in graphene ... have very long mean free paths"*.

"Length and width dependent resistance scaling in GNRs".

"The averaging hopping length between localized states".

Why *"the charge transport is dominated by hopping through localized state"; etc.*

Why **graphene nanotubes**, rolled-up form of graphene, have either conductivity, **metallic** or **semiconducting**.

The rolling-up is realized mainly along two crystallographic directions: along the major axis of anisotropy and in perpendicular to it direction. They define thermodynamically favourable minimum-energy states.

Picture 29

General Conclusion

According to the solutions of the Helmholtz equation, atoms as wave formations have the shell-nodal structure and are molecules of the strongly bound hydrogen atoms.

The main role in the formation of molecules and crystals belongs to the nodal hydrogen atoms.

Chemical bonds are realized in interatomic space along characteristic directions defined by the topology of internodal bonds (along strong bonds realized between the nearby intra-atomic nodes belonging to two external shells each of linked atoms).

Electrons play the secondary role: they define only the strength of chemical bonds.

All possible isotopes (their structure and relative mass) are defined by the extent of filling all potential and potential-kinetic nodes (visible and invisible).

Literature

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- <http://shpenkov.janmax.com/talk2015Gottingen.pdf>

Addition

The disclosed shell-nodal structure of the atoms **represents** something like the “**genetic code**”, which defines, in a definite extent, the structural **variety**, **symmetry**, and **periodicity** in Nature.

In the light of the discovery, a **SEM image** of graphene (as any molecule or crystal) represents, figuratively speaking, the “**top of iceberg**”.