

# Derivation of the Lamb Shift with Due Account of Wave Features for the Proton-Electron Interaction

G. P. Shpenkov

*Institute of Mathematics & Physics, UTA, Kaliskiego 7, 85-796 Bydgoszcz, Poland*

We regard the hydrogen atom as a pulsing spherical-cylindrical wave system to which the three-dimensional wave equation is valid. On this basis, taking into account radial pulsations of the spherical shell of a proton, we arrive at a zero level (background) spectrum of the hydrogen atom. The background spectrum obtained contains the term corresponding to the 2.730 K temperature that is the characteristic value for “relict” background measured by NASA’s Cosmic Background Explorer (COBE) satellite. The frequency gaps 8172.34 MHz and 1057.97 MHz between the nearest background terms among all calculated almost coincide with the most accurate experimental values available for the 1S and 2S Lamb shifts.

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An electron orbiting a proton in the hydrogen atom experiences a slight energy shift. The latter was first found by W. Houston, in 1937 [1], and then measured with high precision in 1947 by W. Lamb and R. Retherford [2]. According to the theory of quantum electrodynamics (QED), the shift is a result of the interaction between an orbiting electron and the teeming virtual particles residing in the surrounding vacuum. Due to fluctuations of the zero field of vacuum, the orbital motion of the electron in an atom is affected to the additional chaotic motion. The main constituents of the energy “splitting”, called the Lamb shift, are the effects of vacuum polarization, electron mass renormalization and anomalous magnetic moment.

Despite of the first and subsequent relatively fortunate calculations made with QED (which was initially developed to explain just the Lamb shift in the hydrogen atom), “It is...far from clear that everything is okay with QED” (M.G. Boshier [3]). Sharing Boshier’s opinion, we assume that not all possible elementary wave processes were taken into account to explain this phenomenon. Now, basing on the *dynamic model of elementary particles* (DM) [4], which is beyond QED, we turn to the explanation of the observed shift on the new basis.

According to the DM, there exist background oscillations of the center of mass of a proton and its wave shell. This oscillations influence the orbiting electron, forming the spectrum of zero level (background) radiation [5]. The background spectrum of the hydrogen atom is derived on the basis of radial solutions for the wave equation in spherical (for the proton) and cylindrical (for the orbiting electron) coordinates. The energetic gaps between the nearest terms of the found background spectrum correspond in value to the 1S and 2S Lamb shifts. We shall now show how one can arrive at the background spectrum resting on the DM.

The hydrogen atom is a paired centrally symmetric proton-electron system. Let us imagine the latter as a pure *wave* dynamic formation. This means that a proton, just like an electron, is in a continuous dynamic equilibrium with environment through the wave process of the definite frequency  $\omega$  (recalling a micropulsar).

A pulsing spherical wave shell of a proton (and of an electron) separates its inner space from ambient wave fields. The shell restricts the main part (core) of the particle from its field part merging gradually with the ambient field of matter-space-time.

Longitudinal oscillations of the *spherical* wave shell of the proton in the radial direction provide an interaction (more correctly *exchange* of matter-space and motion-rest [4]) with the surrounding field-space and with the orbiting electron. The orbital motion of the electron is associated with the transversal *cylindrical* wave field. Therefore, the common three-dimensional wave equation is valid for both cases. Both dynamic constituents of the proton-

electron system have to be described, respectively, by *spherical* and *cylindrical* wave functions.

In the first place, we present here a few formulae which are of a great importance for the derivation of the background spectrum. According to the solutions, electron transitions in atoms depend on the structure of their radial shells. In the *central spherical wave field* of the hydrogen atom, amplitude of radial oscillations of a spherical shell of the proton is

$$A_s = A\hat{e}_l(kr)/kr, \quad (1)$$

where

$$\hat{e}_l(kr) = \sqrt{\pi kr/2}(J_{l+1/2}(kr) \pm iY_{l+1/2}(kr)), \quad (2)$$

$$k = \omega/c. \quad (3)$$

Here  $J(kr)$  and  $Y(kr)$  are Bessel functions;  $\omega$  is the oscillation frequency of the pulsating spherical shell of the proton equal to the fundamental “carrier” frequency of the subatomic and atomic levels [4]. Zeros and extrema of the Bessel cylindrical functions,  $J_{l+1/2}(kr)$  and  $N_{l+1/2}(kr)$  (or  $Y_{l+1/2}(kr)$ ), are designated, correspondingly, as  $j_{l+1/2,s}$ ,  $y_{l+1/2,s}$ ,  $j'_{l+1/2,s}$ , and  $y'_{l+1/2,s}$ . Analogously, zeros and extrema of the Bessel spherical functions are designated as  $a_{l,s} = j_{l+1/2,s}$ ,  $b_{l,s} = y_{l+1/2,s}$ ,  $a'_{l,s}$ , and  $b'_{l,s}$  [6].

The amplitude energy of the pulsing shell takes the following form

$$E_s = \frac{m_0\omega^2 A_s^2}{2} = \frac{m_0\omega^2}{2} \left(\frac{A}{kr}\right)^2 |\hat{e}_l(kr)|^2 = \frac{m_0c^2 A^2}{2r^2} |\hat{e}_l(kr)|^2, \quad (4)$$

where  $m_0$  is the proton mass,  $A$  is the constant equal to the oscillation amplitude at the sphere of the wave radius ( $kr = 1$ ). Let  $kr_0 = z_{l,1}$  and  $kr_s = z_{l,s}$ , where  $z_{l,s}$  and  $z_{l,1}$  are zeros of Bessel functions  $J_{l+1/2}(kr)$ , the following relation is valid between radial shells:

$$r_s = r_0(z_{l,s}/z_{l,1}). \quad (5)$$

The subscript  $l$  indicates the order of Bessel functions and  $s$ , the number of the root. The last defines the number of the radial shell. Zeros of Bessel functions define the radial shells with zero values of radial displacements (oscillations), *i.e.*, the shells of stationary states.

In the *cylindrical wave field*, the energy  $E_c$ , as the sum of energies of two mutually-perpendicular potential-kinetic oscillations of the orbiting electron, is (in the simplest case) equal to

$$E_c = m_e v^2 = m_e \omega^2 A_c^2 = m_e \omega^2 (a/\sqrt{kr})^2 = 2\pi m_e v A_c v, \quad (6)$$

where  $m_e$  is the mass of an electron;  $r$  is the radius of its orbit;  $v$  is the frequency, and

$$A_c = a/\sqrt{kr} \quad (7)$$

is the amplitude, and  $v = \omega A_c$  is the amplitude velocity of its oscillations. Because  $k = \omega/c$ , Equation (6) reduces to

$$E_c = h\nu, \quad (8)$$

where  $h = 2\pi m_e c a^2 / r = 2\pi m_e v A_c$  is an elementary action. If  $kr = \omega r_0 / c = v_0 / c$ , where  $v_0$  is the Bohr velocity, then the amplitude of oscillations  $A_c$  is equal to the Bohr radius  $r_0$ :  $A_c = a/\sqrt{kr} = r_0$ . The constant  $a$ , equal to the oscillation amplitude at the Bohr orbit  $r_0$ , has thus the value

$$a = \sqrt{hr_0/2\pi m_e c} = 4.52050647 \cdot 10^{-10} \text{ cm}, \quad (9)$$

where  $h = 2\pi m_e v_0 r_0 = 6.62606876 \cdot 10^{-27} \text{ erg} \cdot \text{s}$  is the Planck constant.

In a case when exchange (interaction) between spherical and cylindrical fields takes place, the equality  $E_c = \Delta E_s$  is valid. Hence, with allowance for (4) and (8), we arrive at

$$\frac{h c}{\lambda} = \frac{m_0 c^2 A^2}{2r_0^2} \left( \frac{|\hat{e}_p(kr_m)|^2 z_{p,1}^2}{z_{p,m}^2} - \frac{|\hat{e}_q(kr_n)|^2 z_{q,1}^2}{z_{q,n}^2} \right) \quad (10)$$

Thus, we obtained the spectral formula in a general form, where quantum numbers are roots of Bessel functions. Therefore, we can regard these roots as a mathematical variant of spectral terms.

At  $p=q=0$ , zeros of Bessel functions  $J_{0+1/2}(z_{0,s})$  are equal to  $z_{0,s} = s\pi$  [6] and  $|\hat{e}_0(kr_s)|^2 = 1$ , Eq. (10) is transformed into the elementary spectral formula for the hydrogen atom:

$$\frac{1}{\lambda} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right), \quad (11)$$

where  $R = m_0 c A^2 / 2hr_0^2$  is the Rydberg constant.

Because  $R = R_\infty / (1 + m_e / m_0) = 109677.5777 \text{ cm}^{-1}$ , hence

$$A = r_0 \sqrt{2hR / m_0 c} = 9.009369379 \cdot 10^{-13} \text{ cm}. \quad (12)$$

Assuming in the formula (1) that  $kr$  is equal to the first extremum of the spherical function of the zero order, unequal to zero ( $kr = a'_{0,2} = 4.49340946$ ), we find the first maximal amplitude of radial oscillations:

$$\langle A_s \rangle = (1/\sqrt{2})A / kr = 1.417762222 \cdot 10^{-13} \text{ cm}. \quad (13)$$

The center of masses of the proton, performing such oscillations, forms a *dynamic spherical volume with the radius equal to the amplitude of the oscillations and its volume can be regarded as a nucleus*.

Exchange of energy between the proton and the orbiting electron in real conditions occurs, thus, on the background of oscillations of the center of mass of the proton and on the background of exchange with the surrounding field-spaces of a different nature. Hence, the equation of exchange (interaction) should generally be presented as  $E_c = \Delta E_s + \delta E$ , where  $\delta E$  takes into account various perturbations of the orbital electron motion.

The orbiting electron in hydrogen (both equilibrium and exited states) *constantly* exchanges the energy with the proton at the fundamental frequency inherent in the subatomic level  $\omega_e$  [4]. This exchange process between the electron and proton has the dynamically equilibrium character and runs on the background of the superimposed oscillatory field. The latter is characterized by a system of radial standing waves, which define “zero level exchange” [5] in a dynamically equilibrium state of the atom.

The spectrum of frequencies of zero wave perturbation is defined from the equation

$$\frac{1}{\lambda} = R \left( \frac{1}{n^2} - \frac{1}{(n + \delta n)^2} \right), \quad (14)$$

where  $\delta n = \delta r_n / r_0$  is the relative measure of background perturbations  $\delta r_n$  of the orbital radius  $r_0$  (the Bohr radius) at the level of zero exchange.

In the spherical wave field of the hydrogen atom,  $\delta r_n$  depends on the same roots of Bessel functions  $z_{p,s}$  because

$$\delta r_n = \frac{A e_p(z_{p,s})}{z_{p,s}} = \frac{A}{z_{p,s}} \sqrt{\frac{\pi z_{p,s}}{2} (J_p^2(z_{p,s}) + Y_p^2(z_{p,s}))}. \quad (15)$$

Thus, we obtain the following spectrum of waves, generated by zero perturbations of stationary states of the hydrogen atom caused by background oscillations of its mass and wave shell:

$$\frac{1}{\lambda} = \frac{R}{n^2} - R \left( n + \frac{A}{r_0 z_{p,s}} \sqrt{\frac{\pi z_{p,s}}{2} (J_p^2(z_{p,s}) + Y_p^2(z_{p,s}))} \right)^{-2}. \quad (16)$$

On the basis of this formula, we shall estimate a few most probable perturbations of the stationary ( $n=1$ ) and excited ( $n=2$ ) states in the hydrogen atom for the case with  $p=0$  and  $s=1, 2$ , and 3. For this aim, we take the constant  $A$  in the form

$$A = r_p \sqrt{2hR / m_n c}, \quad (17)$$

where  $r_p$  is the theoretical radius of a proton boundary sphere, exactly corresponding to the experimental mass of the proton  $m_0 = 1.67262158 \cdot 10^{-24} \text{ g}$ . The radius  $r_p$  is calculated, in the framework of the DM [4], from the formula

$$m_0 = 4\pi r_p^3 \varepsilon_0 / (1 + k_e^2 r_p^2), \quad (18)$$

where  $\varepsilon_0 = 1 \text{ g} \cdot \text{cm}^{-3}$  is the absolute unit density of matter,  $k_e = \omega_e / c$  with  $c = 2.99792458 \cdot 10^{10} \text{ cm} \cdot \text{s}^{-1}$  and  $\omega_e = 1.869162559 \cdot 10^{18} \text{ s}^{-1}$ . At the level of the fundamental frequency  $\omega_e$ , the proton theoretical radius is

$$r_p = 5.28421703 \cdot 10^{-9} \text{ cm} \quad (19)$$

The hydrogen atom and the neutron represent by themselves two elementary proton-electron systems slightly different in mass. We choose for calculations the system with the heavier mass which has a neutron,  $m_n = 1.67492716(13) \cdot 10^{-24} \text{ g}$ .

The final condition concerns the choice of the numerical factor  $\beta_n$  multiplied by  $\delta r_n$  in the case of  $z_{p,s} = j'_{p,s}$ . The matter is that roots  $y_{p,z}$  represent equilibrium kinetic radial shells, whereas  $j'_{p,s}$  represent extrema of potential shells [7] exhibited under the excitation of the hydrogen atom (note that  $j'_{0,2} = j_{1,1}$ ,  $j'_{0,3} = j_{1,2}, \dots$ ). Hence, for the excited atom, the value  $\delta r_n$  will be slightly differing from the equilibrium value defined by (15). We found empirically that, for the case of  $z_{0,s} = j'_{0,s}$ ,  $\beta_1 = 0.9962$  ( $n=1$ ) and  $\beta_2 = 0.9972$  ( $n=2$ ). Results of calculations by the formula (16) under the above conditions are presented in Tables I - III.

TABLE I. Spectral terms of background radiation of the hydrogen atom, for  $n=1$

$s$	$z_{0,s}$ [6]	$1/\lambda, \text{ cm}^{-1}$ (16)	$\lambda, \text{ cm}$	T, K	$T_{\text{exp}}, \text{ K}$ [8]
1	$y_{0,1} = 0.89357697$	41.693193	0.023985	12.091	
2	$y_{0,2} = 3.95767842$	9.415700	0.106206	2.730	$2.728 \pm 0.002$
	$j'_{0,2} = 3.83170597$	9.688277	0.103218	2.810	
3	$y_{0,3} = 7.08605106$	5.258973	0.190151	1.525	
	$j'_{0,3} = 7.01558667$	5.291609	0.188978	1.535	

TABLE II. Spectral terms of background radiation of the hydrogen atom, for  $n = 2$ 

$s$	$z_{0,s}$ [6]	$1/\lambda, cm^{-1}$ (16)	$\lambda, cm$	$T, K$
1	$y_{0,1} = 0.89357697$	5.212402	0.191850	1.512
2	$y_{0,2} = 3.95767842$	1.177001	0.849617	0.341
	$j'_{0,2} = 3.83170597$	1.212290	0.824885	0.352
3	$y_{0,3} = 7.08605106$	0.657401	1.521143	0.191
	$j'_{0,3} = 7.01558667$	0.662132	1.510272	0.192

TABLE III. Frequency gaps between nearest background terms in hydrogen

$n$	$s$	Terms' difference, $\Delta(1/\lambda), cm^{-1}$ (16)	$\Delta\nu, MHz$	$\Delta\nu_{exp}, MHz$ [9]
1	2	$(j'_{0,2} - y_{0,2})_{n=1} = 0.2726$	8172.34	8172.876(29)
	3	$(j'_{0,3} - y_{0,3})_{n=1} = 0.0326$	978.40	
2	2	$(j'_{0,2} - y_{0,2})_{n=2} = 0.0352895$	1057.97	1057.8446(29)
	3	$(j'_{0,3} - y_{0,3})_{n=2} = 0.0047$	141.85	

We see that at  $p = 0$ , the zero of the second *kinetic shell* [6] is  $z_{0,2} = y_{0,2} = 3.95767842$ ; hence, from (16) it follows that

$$\lambda = 0.106206 \text{ cm}. \quad (20)$$

The zero level of wave exchange (interaction with environment) is not perceived visually and integrally characterized by the absolute temperature of zero exchange. It exists as a standard energetic medium in the Universe. Actually, the wave (20) is within an extremum of the spectral density of equilibrium cosmic background. The absolute temperature of zero level radiation with this wavelength is

$$T = 0.290 (cm \cdot K) / \lambda = 2.730 \text{ K}. \quad (21)$$

The temperature obtained is close to the temperature of “relict” background measured by NASA's Cosmic Background Explorer (COBE) satellite to four significant digits ( $2.728 \pm 0.002 \text{ K}$ ) [8].

The theoretical values obtained for the  $(j'_{0,2} - y_{0,2})_{n=1}$  and  $(j'_{0,2} - y_{0,2})_{n=2}$  differences (Table III) almost coincide with the experimental values for the  $1S$  and  $2S$  Lamb shifts  $L_{1,s} = 8172.837(22) \text{ MHz}$  and  $L_{2,s-2p} = 1057.8446(29) \text{ MHz}$  [9].

Accuracy of the first elementary calculations, presented in Tables, performed on the basis of (16), can be easily improved owing to the relative clarity with the factor which can be exposed to the possible corrections. Actually, only the constant  $A$  (17), defining the oscillation amplitude at the sphere of the wave radius ( $kr = 1$ ), can be changed in (16).

In view of the data obtained, the observation of the cosmic microwave background and the Lamb Shifts provide strong evidence for the existence of zero level radiation of hydrogen (and, apparently, any) atoms in the Universe. They justify in favor of the validity of the background spectrum expressed by Eq. (16) and of the DM [4], which is the basis model for the derivation of the spectrum.

We should note in addition that the words “splitting” or “shift”, strictly speaking, are not appropriate words for the background spectrum (16). All background terms (defined by roots of Bessel functions) exist objectively; therefore, we should speak only about energetic differences (frequency gaps) between them.

Thus for the first time, owing to the DM on the basis of which the formula of the background radiation of the hydrogen atom was derived, the common nature of two phenomena found in the 20<sup>th</sup> century - the Lamb shift and “relict” background radiation (cosmic microwave radiation, CMB) - was revealed.

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